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MIDDLE SCHOOL PRE-SERVICE MATHEMATICS TEACHERS' EXPLANATIONS OF THE FRACTION DIVISION WITH MODELS

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Abstract

This study investigates how middle school pre-service mathematics teachers use the mathematical models in fraction division. A case study research design was used on a study group consists of 37 senior pre-service teachers studying in the middle school mathematics teacher education program in academic year of 2019-2020 at a state university. As a data collection tool, the participants were asked to respond to an open-ended question to analyze how they explain "1/7: 3/4" using models. The results revealed that the pre-service teachers had difficulties in explaining the given operation using models. Further, the pre-service teachers either modeled only the result or represented the dividend and divisor on the model while dividing fractions using models. Some of the pre-service teachers either did not respond to the question or used incorrect models that do not show the meaning of the operation. The obtained results suggest that more specific courses should be incorporated into the curricula of undergraduate mathematics teacher education programs to enhance the pre-service teachers' content knowledge, and the teaching of the concepts that students have difficulty in making sense of should be increased.

Keywords: Content knowledge, division fraction, model, pre-service mathematics teachers.

INTRODUCTION

Fractions, operations with fractions, and their meanings are of vital importance in understanding real-life problems and learning more advanced mathematical topics. It is seen that the conceptual dimension of teaching fractions, especially in the fraction division, is neglected, whereas rote learning is at the forefront (Ma, 1999). Previous studies have reported that teachers and pre-service teachers have misconceptions and limitations about division operation in fractions (Ball, 1990; Işıksal, 2006; Kılcan, 2006; Li & Kulm, 2008; Ma, 1999; Orrill, Sexton, Lee & Gerde, 2008; Seçir, 2017; Tanışlı, Ayber & Karakuzu, 2018; Tirosh, 2000, Yavuz-Mumcu, 2018). Işıksal (2006) noted that although pre-service mathematics teachers could solve the basic questions on multiplication and division of fractions, they did not have sufficient knowledge required to explain these topics and relationships conceptually. Ball (1990) examined the content knowledge and pedagogical content knowledge of pre-service teachers by asking interview questions about three cases of division (e.g. division by fractions, division by zero, and division in algebraic equations), and the results demonstrated that the pre-service teachers only had the knowledge of mathematical procedures, had significant difficulty "unpacking" the meaning of division with fractions, developing representations and reasoning about mathematical operations. Orrill et al. (2008) observed that mathematics teachers' conceptual knowledge about the concept of fraction division was limited, and this might be due to the necessity of in-depth procedural knowledge of operations with fractions. Yavuz-Mumcu (2018) emphasized that pre-service elementary mathematics teachers generally had difficulty in representing fractional operations using models, especially when division operations were involved. The pre-service teachers were asked to explain the operation of "3/5:1/2" using models. It was seen that only 3 out of the 29 pre-service teachers used the model correctly, 21 of them provided incorrect models, and lastly 5 pre-service teachers had partially correct models. The pre-service teachers were found to be able to calculate the result of fractional operations arithmetically in most cases and then struggle to create a model that matched the result they obtained. In addition, pre-service teachers had difficulty in relating whole and fractional parts in the modeling process. Kayhan-Altay and Kurt-Erhan (2017) detected that 43.9% of the 173 pre-service mathematics teachers provided correct models of the division fraction "1/2:1/8" and explained the meaning of the operation. In this respect, the larger fraction (1/2) was modelled first then the amount of small fraction (1/8) was looked for within the model, or 8 times 1/2 was shaded on the model. Bayazit, Aksoy, and Kırnap (2011) measured 35 elementary school mathematics teachers' modelling competencies in representing the fraction division "1/2:1/6". 10 of the teachers clearly represented the divisor in the dividend using fraction strips. It was observed that 4 of the teachers either solved the operation using the invert-and- multiply algorithm and represented the result in the area model, or they had the knowledge about the logic of the operation but could not transfer it to the model. Other teachers either did not respond to the question or drew incomplete models and illogical figures. Further, during the interviews with the teachers, none of the teachers could produce the correct model. In the study conducted by Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992), the pre-service teacher reviewed the division-of-fractions algorithm using the problem 3/4 divided by 1/2 as an example. Subsequently, the pre-service used a picture to demonstrate a division where the divisor was half of the dividend. However, after realizing that she could not draw the model accurately, she solved the problem using the invert-and- multiply algorithm. Kılcan (2006) articulated that in cases where the divisor was not a natural number, the participating teachers did not prefer to explain the operation using models and they generally used models in cases where the divisor was a natural number. It was reported that the most favorite strategy used by teachers was to solve the operations with algorithms. In light of the studies mentioned above, it can be contended that teachers and pre-service teachers have lack of knowledge about representing and explaining fraction division using models. Given that the participants generally preferred to solve the operation using the invert-and- multiply algorithm or represent the result of the operation in the model, it can be implied that they have lack of content knowledge and competency in mathematical modelling instruction.

Given the abstract nature of operations with fractions, especially fraction division, the use of models in transforming the concept into simple ones comes to the fore. Previous studies in the literature have highlighted that models play a crucial role in revealing the meaning of fraction operations and that they have a positive impact on understanding of concepts using models (Alenazi, 2016; Ball, 1990; Kayhan Altay & Kurt Erhan, 2017; Li. & Kulm, 2008; Li & Smith, 2007; Lo & Luo, 2012; Ma, 1999; Orrill et al., 2008; Seçir, 2017; Simon, 1993, Tanışlı et al., 2018; Toluk-Uçar, 2011; Zembat, 2004).

Van de Walle, Karp and Bay-Williams (2020) defined the model as any drawing, object, or picture that represents a concept. They suggested that fractions can be represented by three models: area, length, and set. The area model represents a fraction's part-whole, meaning on a part of a region or area. In the length model, fractions are represented either as a subdivision of a length of a paper strip or as a unit of a given size. For instance, fractions are represented as a length or distance between 0 and a point on a number line. In the set model, on the other hand, fractions are represented based on how many separate elements are in the whole set and in the part (p.381).

The Aim and Significance of the Study

Although it is known that models play a vital role in learning fractions and operations with fractions, the use of models in teaching processes has been neglected (Çelik & Çiltaş, 2015). For this reason, pre-service teachers, who are future teachers, need to have in-depth knowledge and experience of the use of models in teaching fraction operations to leverage their effective instruction. Additionally, a series of studies (Bayazit et al., 2011; Borko et al., 1992; Kayhan-Altay & Kurt-Erhan, 2017; Orrill et al., 2008; Yavuz-Mumcu, 2018) provided insight into pre-service teachers' and teachers' explanations of the use of models with respect to the fraction operations (e.g. cases where a large fraction is divided by a smaller fraction, divisor fraction is a fraction, and the denominators are multiples of each other or the result is a natural number). Given the previous studies in the literature, the pre-service teachers' performance when referring to fraction division using models, dividing a small fraction by a larger fraction, and in cases where the denominators are not multiples should be identified. For this reason, the present study attempts to examine how pre-service mathematics teachers explain the division of a small fraction by a larger fraction using modelling.

METHOD

In this research, a qualitative case study was performed. The objective of the qualitative research is to get in-depth details as much as possible about an event, person or process (Merriam, 1998). In a case study, which is one of the qualitative research types, a "how" or "why" question is being asked regarding a contemporary set of events which the investigator has little or no control at all (Yin, 2009). Since the present study sought to elaborately investigate how the pre-service teachers explained fraction division using modeling, this method was employed.

Study Group

The participants of the study consist of 37 (32 female, 5 male) senior pre-service mathematics teachers enrolled at a state university in Turkey in the fall semester of 2019-2020. Given that the pre-service teachers are required to know the expectations and teaching methods of the concepts in the curriculum, the study group included those who successfully completed the Special Teaching Methods I-II courses in the first place.

Data Collection Tools

The Fraction Division Test (FDT) which was developed in a project to assess pre-service teachers' content knowledge of division fraction was used as a data collection tool. In this test pre-service teachers were asked to explain how they divide i) a fraction by a natural number, ii) a natural number by a fraction, and iii) fractions by fractions using models. For the reliability and validity of the open-ended questions in the FDT, the opinions of one teacher and one faculty member who are experts in mathematics education were taken, and the questions were reviewed in terms of language and content before initiating the implementation. The pre-service teachers were generally able to correctly model the open-ended questions in the first two items. However, the responses given to the open-ended question in the third item are quite different from the responses in the other items. Therefore, the present study only focuses on the written statements of the pre-service teachers, which indicates how pre-service teachers explain the division operation "1/7: 3/4" using models.

Data Analysis

A descriptive analysis was used to identify the quality of the pre-service teachers' responses, and the responses were coded as correct, incomplete, and incorrect. During the coding process of the responses, the indicators provided in Table 1 created by the researcher and the coder were taken into account.

Table 1. Indicators used in coding responses

Code	Indicator
Complete	• Creating complete and accurate mathematical models (area, length, set, number line models) that indicate the meaning of the operation
Incomplete	• Unable to create a model that represents the fraction operation using models (area, length, set, number line models) • Writing the algorithm to perform the operation and creating a model (area/number line models) according to the result of the algorithm
Incorrect	• Writing the algorithm to perform the operation and creating a model (area, length, set, number line models) according to the result of the algorithm • No written statement

While developing the indicators in Table 1, the pre-service teachers' responses were examined, and the common opinions were determined. The written responses of the pre-service teachers were analyzed separately by the first researcher and an expert in the field. Upon the completion of the individual analysis, the analysis was shared, and discussions were held whereby it was intended to resolve disagreements. Afterwards, all the data were reanalyzed individually by the researcher and the expert. The reliability formula (Reliability = Consensus / (Agreement + Disagreement)) proposed by Miles and

Huberman (1994) was employed and the inter-coder reliability was found as 91%. In addition to that, disagreements were discussed, and the analysis was finalized. Given that Miles and Huberman (1994) suggest the reliability of the coding should be at least 80%, the data has a very high level of reliability.

RESULTS

This section presents the main findings obtained from the pre-service teachers' written responses to the open-ended question in which they explained the "1/7: 3/4" fraction operation using models. The results denoted that pre-service teacher had the most difficulty in dividing a small fraction by a larger fraction using models. For this reason, this study examined the models used by the pre-service teachers for the "1/7: 3/4" operation. Table 2 summarizes the findings on the pre-service teachers' performances of using models while explaining fraction division.

Table 2. The pre-service teachers' performance of using models while explaining the fraction division

	Area Model	Number Line Model	Total
Correct	-	-	-
Incomplete	25	2	27
Incorrect	3	-	3
Blank	-	-	7
Total	28	2	37

As can be seen from Table 2, none of the pre-service teachers provided a correct model while explaining the given fraction division operation. While explaining the fraction division, 25 of the pre-service teachers using the model benefited from the area model, and only two pre-service teachers used the number line. On the other hand, 10 pre-service teachers either did not respond to the question or wrote the following statement: "I couldn't do it" or used an incorrect model.

27 (72.9%) of the pre-service teachers generated representations that were interpreted as incomplete modelling. For instance, given PT7's response when referring to the operation using the area model in Figure 1, PT7 represented the dividend fraction (1/7) in the first stage and subsequently 1/7 was divided by 3/4 in the second stage, yet there was no representation for the result. Further, the pre-service teacher did not provide any algebraic solution indicating the result of the operation.

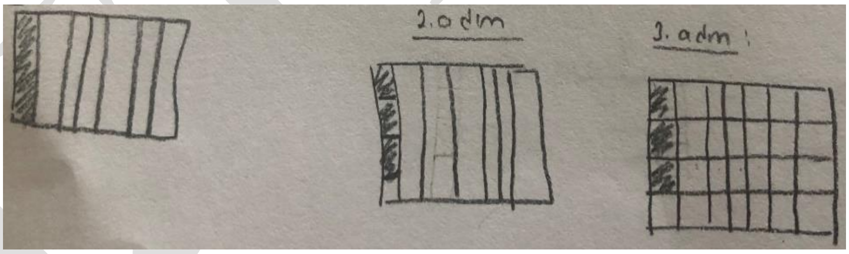

Figure 1. Incomplete use of area model by PT7

Figure 2 illustrates representation of PT10. In a similar vein, the PT10 represented the dividend fraction first and then looked for the divisor in the dividend. Using the area model, the pre-service teacher attempted to represent the operation via arrows yet failed to provide the correct model. PT10 did not complete the modeling but only wrote the result of the given operation.

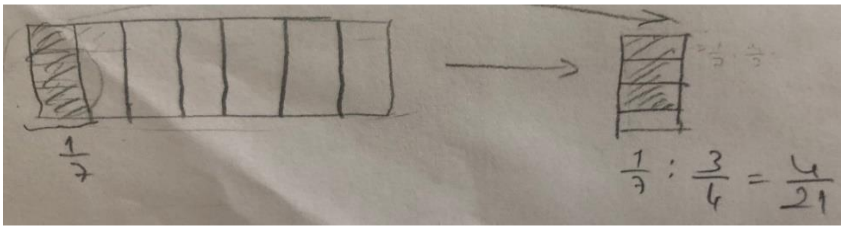

Figure 2. Incomplete use of number line by PT10

Only 2 (5.4%) of the pre-service teachers attempted to represent the operation using the number line. However, looking at the representations, it is seen that the pre-service teachers (PT17, PT20) generated representations similar to those who provided incomplete models for the given operations. For instance, as seen in Figure 3, the fraction operation was represented on the number line by PT20.

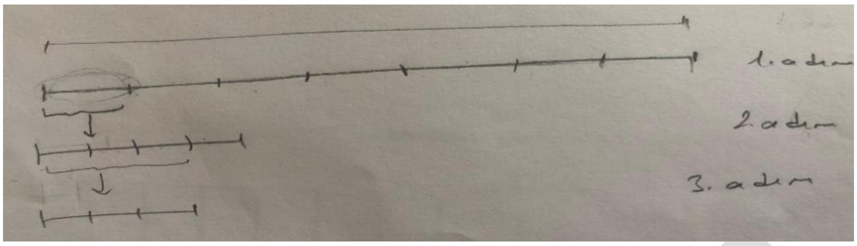

Figure 3. Incomplete use of area model by PT20

PT20 first divided a whole into seven equal parts and marked the first part as 1/7. Then PT20 divided the fraction 1/7 into four equal parts and marked the part as 3/4. However, the pre-service teacher did not show the operation, pre-service teacher only marked the divisor on the dividend.

12 (32.4%) of the pre-service teachers who had incomplete modelling generated representations, implying that the result was modeled without providing any meaningful justification. Some of the pre-service teachers either modeled only the result of the operation, or they performed the operation algebraically via the invert and multiply algorithm or the common denominator algorithm and then represented the result using the area model. For example, as given in Figure 4, PT5 only modeled the result without showing the operation.

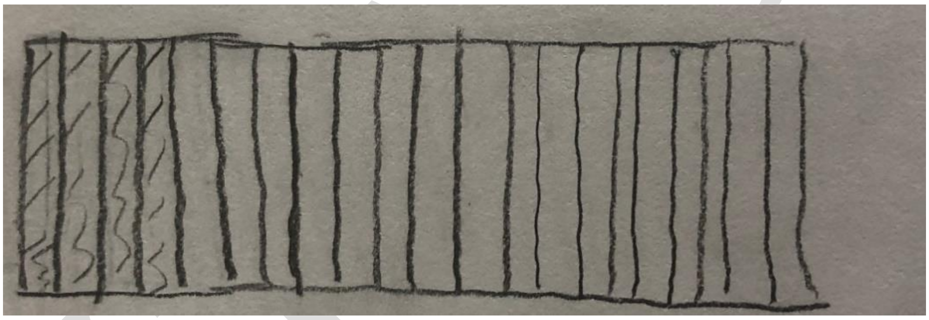

Figure 4. Incomplete use of area model by PT5

The PT5 divided a whole into 21 parts and marked 4 parts and did not have any explanation for fraction division. Another pre-service teacher (PT25) gave incomplete modelling, as depicted in Figure 5.

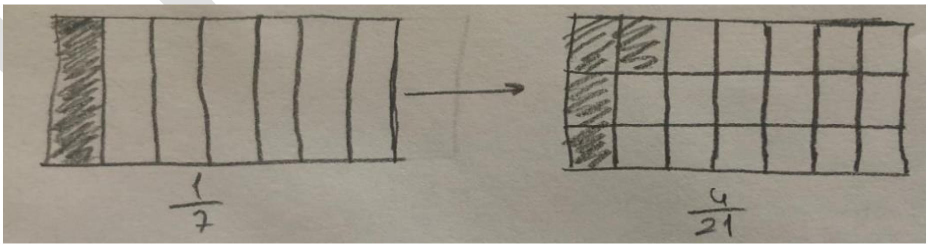

Figure 5. Incomplete use of area model by PT25

Given PT25's response in Figure 5, it is seen that PT25 first represented the dividend fraction and then modeled the result. The pre-service teacher adopted a result-oriented approach, and the result was represented in the area model. As a result, PT25's response was coded as incomplete modelling.

Given the responses of the pre-service teachers who provided incomplete modelling, two cases emerged. In the first case, it was seen that the pre-service teachers had knowledge about the meaning of the fraction division operation, but they could not achieve the result while representing the operation using modeling, and in the second case, they were inclined to represent the result of the operation on the model.

10 (27%) of the pre-service teachers' responses were categorized as incorrect modeling. Seven of these responses (18.9%) either did not respond to the question or made the following remark: "I could not do it". Given the remaining responses, the pre-service teachers could not create a model for explaining the operation. For instance, Figure 6 depicts incorrect use of the area model by PT26.

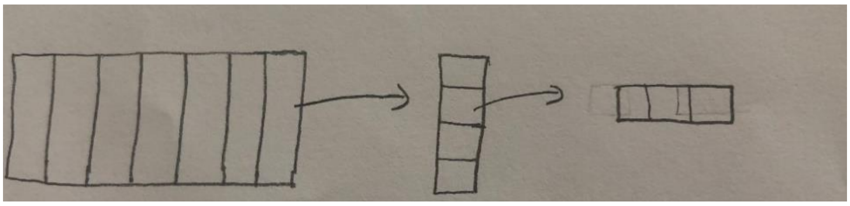

Figure 6. Incomplete use of area model by PT26

As can be seen from Figure 6, PT26 divided a whole into seven equal parts, then divided a part into four equal parts, and the area model with three equal parts was created. The area model used by the pre-service teacher while representing the operations does not give a clue about the meaning and solution of the operation. Consequently, PT26's response was evaluated in the context of the incorrect modelling category.

Given the responses evaluated in the incorrect modeling category, the pre-service teachers either did not express their opinions or could not provide the correct use of a model while solving the operation. As a result, none of the pre-service teachers could explain the given procedure using the correct modeling. The findings of the study reveal that the pre-service teachers are insufficient in explaining the division of the small fraction by the large fraction with models.

DISCUSSION and CONCLUSIONS

This study sought to examine how pre-service teachers represented division of a small fraction by a larger fraction using models. It was observed that the pre-service teachers were lack of knowledge in explaining the given operation using modelling. In the literature (Van de Walle et al., 2020) it is reported that operations with fractions can be represented by area, length and set models. In the study, pre-service teachers used only area and length (number line) models. The pre-service teachers using these models were able to represent dividend and divisor fractions, however, they could not explain the operation process and represent the result of the operation on the model. Some of the pre-service teachers tended to indicate only the result of the operation on the model. It was found that ten pre-service teachers either did not respond to the question or provided incorrect modeling. Given the results, it can be implied that pre-service teachers lacked the required knowledge to explain a fraction division using modelling. In parallel with the findings of the present study, Kılcan's (2006) study concluded that teachers had more difficulties in creating models in fraction division and preferred to use more models when dividing a natural number into a fraction or a fraction into a natural number in the teaching process. The current study also discovered that the pre-service teachers' performance in explaining the operation stated in FDT (e.g. dividing a natural number into a fraction and a fraction into a natural number using modelling) was higher than the dividing fractions, especially dividing small fractions by large fractions.

In a similar vein, Yavuz-Mumcu (2018) argued that the pre-service elementary mathematics teachers mostly had difficulty in demonstrating that they looked for the divisor in the dividend, especially while dividing fractions using models, and they represented the result of the operation on the model. Bayazit et al. (2011) also asserted that teachers could not produce the models accurately when dividing fractions by fractions, and they were generally competent in representing the divisor in the dividend fraction in the models they created. When the pre-service teachers' responses to the open-ended questions were examined, it was seen that the teachers and pre-service teachers had difficulties in explaining the operations in which a smaller fraction was divided by a larger fraction, and the denominators were multiples of each other, using modelling. According to Kayhan-Altay and Kurt-Erhan (2017), approximately half of the pre-service teachers used the correct model in the operation in which a large fraction was divided by a smaller fraction and the result was a natural number. Consequently, the present study yielded consistent results with the relevant literature. The results denoted that the-pre service teachers exhibited lower performance in dividing the small fraction by the large fraction using models.

Previous studies in the literature revealed that models played a major role in revealing the meaning of operations with fractions and that models and concepts had a positive impact on making sense of the concepts (Alenazi, 2016; Ball, 1990; Kayhan Altay & Kurt Erhan, 2017; Li & Kulm, 2008; Lo & Luo, 2012; Ma, 1999; Seçir, 2017; Smith, 2007; Tanışlı et al., 2018; Toluk-Uçar, 2011; Zembat, 2004). In addition to that, as reported in the literature (Bayazit et al., 2011, Çelik & Çiltaş, 2015, Orrill et al., 2008) that although it is known that models are important in learning fractions and operations with fractions, the use of models has been neglected in teaching processes.

It is apparent that both pre-service teachers and teachers do not have in-depth knowledge of the use of models in teaching fraction operations. Thus, efforts should be made to improve pre-service teachers' content knowledge. With this in mind, activities that promote the effective teaching of both fraction operations and other concepts should be placed into the undergraduate course content, thereby enhancing pre-service teachers' content knowledge and pedagogical content knowledge. Further studies can dwell upon different concepts whereby teachers' and pre-service teachers' levels of content knowledge and pedagogical content knowledge for teaching can be analyzed and their shortcomings can be addressed. In other words, studies should be conducted to support pre-service teachers' development.

Ethics and Conflict of Interest

The authors acted in accordance with the ethical rules in the research. The authors declare that they have no conflict of interest.

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