MISCONCEPTIONS IN GEOMETRY AND SUGGESTED SOLUTIONS FOR SEVENTH GRADE STUDENTS

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ABSTRACT

The principal aim of this study is to find the weaknesses of secondary school students at geometry questions of measures , angles and shapes , transformations and construction and 3-D shapes. The year 7 curriculum contains 4 geometry topics out of 17 mathematics topics. In addition to this , this study aims to find out the mistakes, 28 , 7th grade students made in the last 4 exams including two midterms and two final exams.To collect data, students were tested on two midterms and two final exams using open–ended questions on geometry to analyze their problem solving skills and to test how much they acquired during the year.Frequency tables were used in data analysis.To fulfill this aim in the first midterm exam the subject measures were tested.In the first final exam which followed the first midterm exam in addition to measures and angles shapes skills were also tested. Following these tests , in the second midterm we tested the students on transformation and construction. A descriptive methodology and student interview were used in the study to analyze and interpret the results. The results from this study revealed that 7th grade secondary school students have a number of misconceptions, lack of background knowledge, reasoning and basic operation mistakes at the topics mentioned above.

Keywords: mathematics education, student difficulties, geometry questions, misconceptions, geometrical errors, teaching suggestions for geometry.

INTRODUCTION

The general aim of mathematics is stated as making an individual acquire the mathematical knowledge needed in daily basis , teaching how to solve problems , making him/her have a method of solving problems and acquiring reasoning methods (Altun,2008) .For this purpose to acquire mathematical concepts one should be able to visualize the diagrams.In other words, mathematics is the field in which preconditions are crucial so before the teaching process student backgrounds on the subject should be tested (Baykul , 1987).Gagne divided the concepts into two as concrete and abstract concepts.Concrete concepts are learnt starting from the beginning of life by the person himself.However to learn abstract concepts sometimes being taught by others is necessary (Senemoğlu,2000).In this context, mathematics based learning should be done according to three aims listed above (Baykul,2002).

- To student acquiring mathematical concepts.
- To understand mathematical operations.
- To help students make connections with the concepts and operations.

While analyzing the literature of mathematics , it was found out that students have difficulties learning concepts and their interconnections. (Tall ve Razali, 1993; Thompson, 1994; Stacey ve MacGregor, 1997; Simon vd, 2004; Inzunza, 2006; Ben-Hur, 2006; Chiu ve Klassen, 2008). However, the teaching and learning mathematics in schools is still dominated by teacher-centered and textbook oriented approach (Indradevi, 1998; Lim & Hwa, 2007). There is minimal use of visualization tools such as the Dynamic Geometrical Tool and graphing tools in mathematics classroom (Pumadevi, 2004). One of the reasons is because of the teachers' attitude and beliefs about mathematics and the use of these visualization tools. Another reason is because of limited skills of using these tools (Handal, et. al 2004).

The origin of the word geometry is earth measure. Geometry was first used for agriculture and construction purposes. Egyptians first used Pythagorean theorem to determine a square corner for a field or the base of a pyramid. Then empirical and quantitate geometry was transformed by Greek scholars from sixth through the fourth BC to logically order body of language. Learning geometry is not just learning the definitions or the attributes of geometrical concepts but also to have the ability of analyzing the properties of two (2D) and three dimensional (3D) geometric shapes and develop

mathematical arguments about geometric relationships, to specify locations and spatial relationship, to apply transformations and to use symmetry, visualization, spatial reasoning, and geometric modeling to solve problems (NCTM, 2000).

Research also indicates that children prefer to rely on a visual prototype rather than a verbal definition when classifying and identifying shapes ([Gal and Linchevski, 2010] and [Van Hiele, 1986]). Specifically, when a child holds both a verbal definition and a visual prototype for a given geometric concept, the child often calls upon the visual prototype rather than, or in spite of, the verbal definition when assigning class membership. For example, Fischbein and Nachlieli (1998) note that although students could give the correct definition of a parallelogram, many relied on the visual prototype instead of applying their definition when identifying shapes. Similarly, Archavsky and Goldenberg (2005) find that the interaction between formal definitions and mental images of geometrical figures are often in conflict. This suggests that students are not necessarily using those "correct" of "formal definitions" as mathematical definitions, but as *descriptions* of the elements in their category of, say, 'parallelogram. According to Piaget and Inhelder(1956), there are certain stages of learning starting from birth. These stages are
Stage 0: scribbles (less than 2)
Stage 1: topological - irregular closed curves to represent circles, squares,etc(2-4 years)
Stage 2: projective- progressive differentiation of Euclidean shapes (4-7 years)
Stage 3: Euclidean- ability to draw Euclidean shapes(7-8 years)

Although there are specific age groups in this, it has not been widely accepted. It has been suggested that even younger children can sometimes operate with some Euclidean concepts. It is probable that topological, projective and Euclidean notions all develop over time and their usage becomes increasingly integrated. Piaget suggested that children looked at the world from a very different perspective than adults did. So scientists started to investigate the reasons behind it by listening carefully what students were saying and doing on a variety of subject-matter tasks. They found surprising facts that students acquire ideas that completed often quite effectively with the concepts presented in the classroom environment. They had a powerful development of conceptions but they were sometimes inconsistent with the accepted mathematical and scientific concepts.

The Van Hiele model (1986) continues to be the best-known theoretical account of students' learning about shape. The model suggests that children have to take a sequence of levels in a fixed order in their learning about shape. The first three levels in the model are as follows: the *Visualization* level (Level 1, also known as the level of *recognition*) in which students recognize and learn to name certain geometric shapes but are usually only aware of shapes as a whole, and not of their properties or of their components; the *Analysis* level (Level 2, also known as *descriptive*) students begin to recognize shapes by their properties; the *Abstraction* level (Level 3, also known as *relational*), students begin to form definitions of shapes based on their common properties, and to understand some proofs. In other words, the establishment of family resemblance must be definition-driven, which, according to linguists such as Rosch, is not the way that categories are usually formed. Perhaps not surprisingly then, research has shown that most students do not make it past this stage in their school education (Clements & Battista, 1992**).**

Many teachers have observed that many young children have numerous misconceptions about geometry When a teacher discusses a geometry proof problem in class, it generally involves oral presentation of a formal proof and body movements pointing at different parts of the figure of the problem. Students must watch, listen, jot notes, and think as a lecture proceeds. They have to refer to many elements of the instruction and incorporate them into their memory (Sweller, 1988). This often causes cognitive overload and poses a negative effect on students' learning. Numerous researchers have experimented different ways of teaching and found serious problems in geometry learners: incomplete comprehension of the problem and mathematical symbols, producing proofs based on direct visual elements (e.g., Chazan, 1993; Healy & Hoyles, 2000), lacking strategic knowledge in producing proofs, etc. Addressing the difficulties in learning geometry, Duval (1998) and Healy and

Hoyles (1998) explained that geometry instruction is often more complex than that of numerical operations or elementary algebra.It is therefore more important that geometry instructions incorporate new and tested approaches such as using visual and multimedia tools in the classroom.

Studying geometry is an important component of learning mathematics because it allows students to analyse and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics. Therefore , students need to develop an understanding of geometric concepts as well as gaining adequate geometry related skills.In this project, analyses the development of geometric skills and the use of tools ,reproduction of constructions, properties verification ,conjecture and research.It can be said that geometry is not used by students from the beginning due to their previous static learning experiences. Another difficulty with some of the students is the geometric language comprehension. After this survey , a seventh grade teacher can analyze students' geometric mistakes and help them to improve their geometric knowledge.In this paper , we describe some guided research techniques for teachers of seventh grade students' in a geometry lesson.This article gives the techniques about teaching .

In our sample class, When construction activities are used, they involve developing new ideas and connecting these with students' existing ideas. If students are not in a particular level of Van Hiele model they might not be able to perceive what the teacher sees in a geometric situation so higher levels of understanding is required. Misconceptions arise frequently if learners bypass or skip a level from the model. A teacher should get students to explain how they come to their answers or rules so that s/he can analyze the faulty interaction between the students' extant ideas and the new concept. By this way the teacher can understand the reason behind misconceptions and they can be corrected by challenging or contrasting it with the right conception. Students' prior learning sometimes arises misconceptions either in the classroom or from their interaction with the social and physical world. However the search for the origins of those misconceptions can not be located to the root of an educational problem. If misconceptions are persistent and resistant to change, that means they have got strong experiential foundations.

The Aim of the Study

The aim of this study is to reveal the performances of 7th grade college students at geometry and to show the conceptual difficulties they face while learning . By doing this , the study tried to identify the misconceptions which arouse during the learning process of geometry.

SAMPLE

28 seventh grade students consisted of 12 males and 16 females at Turk Maarif Koleji in Cyprus.

METHOD

The porpose of this research is to determine college students'misconceptions on geometry subject . The descriptive methodology and student inteview were used in the study to analyze and interpret the results.The descriptive method was used since the main purpose of this study is to clarify an existing situation .This descriptive research analyzed the perspectives and experiences of 28 students' exam papers (two midterms and two finals) .Students' recognition of certain shapes are sometimes affected when they develop a concept image or its properties because they fail to identify the examples of shapes because of the reason that they can't visualize the images of figures.

The Importance of Technology in A Geometry Class:

Technology enables both students and teachers to access wide range of tools to use in mathematics. Perkins (1995) offered three stages in the process of understanding in the context of an information and communication technology. These are

- They offer students explanations
- Make relational knowledge available
- Students can possess revisable and extensive web explanations

Problem

The main problem addressed by this of research is the reality of misconceptions that the students already passes or acquire during geometry lessons . These misconceptions are often related to shape perception and three dimensions.

Student Interview Part:

Students are interviewed and askedfive questions to have their opinions taken. The researcher recorded the face to face interviews by taking notes. In order not to create a disturbing environment no recording machine was used. In the students' statements above, the students' names are represented by the numbers in the parenthesis. Main subjects of the research and the data of 10 students, which were above randomly chosen 7th grade students, at the end of the term, are:

1. What do you think about Geometry Lessons? Do you find it interesting?

Eight out of ten students love Geometry Lessons and they are interested in the lesson. One of these ten students loves Geometry Lessons however s/he finds the measurements subject challenging. S/he prefers subjects which includes logic and operations. The other 1 accepts the fact that Geometry is necessary but since it is very time consuming, it is not interesting. According to this data, 80 percent of students said they loved Geometry Lessons. Although the students love the lesson, they make mistakes so it can be resulted that more quizzes should be done and student misconceptions need to be more emphasized after the quizzes and addition to this more thought provoking questions should be chosen and the subjects should be related to real life.

2. Is there enough time to teach students how to draw geometrical shapes?

Seven students out of ten said that the time spent on geometrical shape drawing is enough. Two of them said that more time can be spent. The last student said that the time spent on drawings is not enough. According to this data, seventy percent of observed students think the time spent on drawing objects is enough. If enough time is spent, the other likely consequence of student misconceptions may be the method of the teacher. More colourful materials or computers on drawings can be used to get the attention of the students.

3. Do you want to change Geometry lessons into something more visual by using computers?

Nine students said that computer use can make the lessons more interesting. It was thought that using computers enable them to visualize and this helps them to learn permanently. One of the students thinks that it is not necessary to use computers. S/he thinks that teacher drawing on the board is more helpful. According to this data, ninety percent of the interviewed students think that computer use in Geometry lessons helps them to learn and remember better.

4. How do you learn and remember the rules and formulae?

Two of the students said that they learn them by writing. One of them said by memorizing. Four of them said by writing and visualizing. One said that by writing and drawing. One said by reading out loud and writing. The ones, who learn by visualizing, claim that when they write and stick papers to some places around the house like their study or wardrobe doors find it easier to learn the rules and the formulae. The last one student said that s/he learns by making logical connections with them and so that she remembers better later by the logic she has created. According to the data, teachers should give students choices of learning rules and formulae so that the students can choose the best for them.

5. Why do you think Geometrical mistakes are made in the exams?

Seven of the students think the cause of the mistakes is hastiness and negligence. One of the student said that the classrooms are crowded so students can not get enough attention from the teachers. One of them said that s/he underestimated drawings so when s/he saw them in the exam s/he was panicked and could not do them properly. The last one thinks s/he did not study enough so got confused in the exam. According to the data, since seventy percent of the students think that the cause of their mistakes are negligence and hastiness, the teachers can suggest them methods to avoid their habits.

The data received after student interviews are directly quoted. During the interviews, it was found that students are satisfied with the method used however teachers can use computers and give more focus on drawings.

Direct Student Opinions

Student (1): '....I find Geometry lessons enjoyable however more emphasis should be given to drawings and more assignments should be given. I had the most difficulty in the enlargement topic.....'

Student (2): '....I didn't have any difficulties in geometry. After the mathematics topic algebra, it gave me motivation. I enjoyed drawing, using the compass and the protractor. I learn the formula by writing them on papers and white board....'

Student (3): '....There should be more focus on student mistakes. I was motivated in the lesson but colours blue and green should be used more since they are loved more by students and they can make the drawings catchier. I write formula on papers and stick them on my study table so that whenever I look at my table I see them and it would be more difficult for me to forget.

Student (4): '…..If I can relate the subject about something funny with my life I can remember it…..' According to the data received from student interviews it can be concluded that: Teachers should show more emphasis on objects and drawings, create relations in between geometry subjects, bring more daily materials into the classroom, find more thought provoking questions, create a more student centered environment, concretize the abstract concepts and use more visuals and exercises.

Table shows the list of concepts which should be addressed throughout the curriculum.

SEVENTH GRADE TOPICS			
MEASURES	**ANGLES**	**TRANSFORMATIONS**	**CONSTRUCTION & 3-D SHAPES**
Metric measure	Properties of a triangle	Transformations	Constructing triangles 1
Imperial measure	Angles in parallel lines	Combinations of Transformations	Constructing triangles 2
Perimeter and area of a rectangle	Properties of a quadrilateral	Symmetry	Bisectors
Area of a triangle	Properties of a polygon	Enlargement 1	Constructing perpendiculars
Area of a parallelogram and a trapezium	Congruent Shapes		Loci
	3-D shapes		Surface area of a cuboid
			Volume of a cuboid

Fırst Midterm Exam

The first midterm exam consisted of 25 questions.3 out of 25 questions were on geometry subjects

Table 2: First mıdterm exam geometry mısconceptıons

Mistake Made	Possible Reasons	Suggestions
While the area of the triangle was found the student forgot to divide the number by two which was on the area formula.(The area formula of triangle is base times height over two and the student forgot dividing it into two)	• Just memorized the formula • Cant visualize the image • Lack of reasoning • Few authentic in the primary grades	• More exercise on the topic • Frequent use of images by more interactive teaching • More visual –object use • Deduction of the area formula in class
Operation mistakes while finding the shaded area from the total	• Lack of spatial/thinking • Lack of construction idea • Lack of background education on operations	• More exercise • More homework • Practising the same procedure on paper to make understanding easier • More practice should be done on operations during primary school
Wrong or missing formulae use.(ex: area of parallelogram is base times height.The student divided base times height by two)	• incomplete understanding • No concentration • Not enough practice of the topic	• Computer based teaching can be used to show students the formulae in more fun and colourful way to make them remember easier.

First Midterm Final Exam Results

In the first midterm final exam , there are 25 questions of which 8 of them are on geometry subjects.

Table 3: The first midterm final exam misconceptions

Mistakes Made	Possible Reason	Suggestıons
Wrong formula use (area of triangle , parallelogram..etc)	-Can not understand the term area. -no proper understanding of the formulae.	-More warm –up before teaching about shapes -Ask students to find the shapes in their real lives. For example a square coffee table , a rectangular notebook, triangular ashtray so that they can understand the shapes better.
No given reasons for the answers	-Problem in the second language usage(can not express themselves in the language) -Can not give explanations to their answers	-More mathematical term use in the classroom -More practice -More stress on explanations in the classroom.
Lack of assimilation of the angles in parallel lines such as alternate and corresponding angles	-Ignore the importance of angles in parallel lines	-The importance of looking at the angles should be emphasized more in the classroom -Colourful images can be used to show alternate and corresponding angles.
-Lack of recognition and of perception properties of quadrilaterals -Can not distinguish the types of quadrilaterals	-Can not visualize -Can not assimilate the properties of quadrilaterals -Students put in little effort.	-More emphasis on properties of quadrilaterals and the similarities and differences while teaching -The students should be involved more during the similarity and difference stages of learning
Wrong conversion of metric measurements (such as changing millimetre to metre)	-No adequate use of conversions in real life -Not enough practice or studying	-Variety of activities can be used in the classroom to show their use in real life (such as showing the metric system on their own rulers)

Wrong detection of angles in an isosceles triangle	-Can not connect the background data learned in primary school to new material. -Aswering the questions spontaneously without reading the rubric of the question. -Lack of spatial reasoning	-More variety of questions on different types of triangles
Operation mistakes (during area calculations , multiplication,addition,subtraction or division mistakes).	-Lack of concentration -Underestimate the importance of operations	-Lectures can be given by student advisors on paying attention tecniques -More practice on operations
Mistakes done on angle , side and parallelism properties on special quadrilaterals	-Insufficient practice and learning -Can not assimilate the properties of quadrilaterals	colurful materials can be used while teaching properties of quadrilaterals to show the equal angles and sides and parallelism.(equal angles can be shown in red and equal sides in green to emphasize the difference)
Can not distinguish the concepts of equations and expressions (for example when the side lengths are given in algebraic expressions , students are unable to find the area)	-Equations and algebraic expressions topic are not learned well.	-Instead of using x,y,z (which are frightening letters for students a,b,c can be used more to show the unknown)

Table 4: The second midterm exam misconceptions

Mistake Made	Possible Reason	Suggestions
-While the student was doing enlargement s/he didn't write the coordinate of the center of the enlargement	-The student read the question carelessly -The student didn't follow the instructions of the question carefully	-Teacher should emphasize the importance of reading the questions more carefully to give relevant answers. -The teacher should summarize the topic to improve the understanding of students
-The students mixed the names of three dimensional objects. For example: instead of writing cuboid, the student wrote cubic.	-Basic vocabulary mistake The student started with wrong step so s/he couldn't finish correctly.	-Details should be shown clearly in the classroom -Revisions and more practice should be done
-The student found the sum of the interior angles incorrectly and also found the size of each interior and exterior angles incorrectly	-Learning formulas and definitions inadequately. -Students don't know what to do properly.	-Students should be encouraged to study and practice harder
-The student shifted the lines while applying reflection and rotation.	-Student can't use the tracing paper properly and counts the squares on the paper incorrectly.	-The use of tracing paper should be shown in detail by using computer based teaching

Table 5: Second Term Final exam misconceptions

There are 20 questions and 8 of them are geometry topics.

Mistakes Made	Possible Reasons	Suggestions
Missing information in descriptive questions(for example, when the student was asked to describe transformation s/he did not use specific words like translation, rotation)	• Lack of enough knowledge • Forget the details about the topic	• Students should be more careful in the exams while describing transformation • The teacher should revise transformation more in detail from beginning to end by using visual aid.
The student did mistakes while enlarging objects and wrong use of coordinates	• Did not understand the process of enlarging. • Confused the coordinates when enlarging objects	• Computer based exercises can be practiced so that students can get a better knowlwdge on the

		enlargement topic. • Describe the position and movement more in detail. • 2 and 3 dimensional objects should be used to make students able to visualize the images.
The student multiplied the number by 2 instead of 3 when calculating the volume of a cube	• Did not know the formula • Confused the formula of the cube with the formula of area	• To teach students volume , use visuals of 3-d objects from different perspectives and analyze the idea of volume
Wrong use of vocabulary (for example the student used the word "translation " istead of "transformation"	• Lack of recognition of formal terms	• Formal mathematical terms should be emphasized more during teaching and practising
Measurements were wrong in beraing questions	• Wrong use of protractor • Wrong application of angle rules	• Make students use of real life objects in the classroom so that they can visualizeand learn better
Questions on polygons and square ,the student considered just one angle of the polygons and ignored the square	• Can not analyze compound figures	• Use geometric images to give students the idea of combining objects
Could not draw angle bisector (for example the student was expected to draw the angle bisector of the given angle but s/he constructed just the angle)	• Misunderstood the questions	• More variety of questions on similar topics should be practiced to avoid misunderstandings
Cube's surface area was miscalculated because of wrong length measurement	• The connection between the length and surface area calculations was not understood properly	• Strategies should be developed to teach students perceptions of cube and cuboid such as using related visual examples to answer questions of surface area and volume
Wrong rotation of images	• Wrong use of tracing paper	• Make students recognize positions and directions . • Use colorful images to teach students rotating objects • Make students draw the shapes onto their own notebooks and ask them to interpret the rotation.
Wrong positioning in translation questions	• Insufficient concentration • Mislearned measuring	• More practice at their own pace • Teacher should make available more practice questions (Moodle can be used more frequently and efficiently)

Second midterm final exam

If the proper terminology is used in context verbally when introducing new terms in geometry, students become familiar with the written solutions and this helps them to develop and learn the necessary vocabulary. The teacher can also insist on their using it as much as possible and associate the new terms with diagrams, representations and symbols so that students can easily connect them with the newly presented topic. Students have a lot of misconceptions while they are using the protractor so the teacher should always warn the students that they should always place it in the correct position when they measure an angle. The instruments and their features in mathematics and geometry should always be introduced properly. Proper use, sufficient practice and formative assessments guaranties their familiarity with the instruments and enables them to use them correctly.

When shapes are in a non standard position they are difficult to be identified. For example if they are not upright or in their usual position, common shapes are not recognized.

What is achieved at the end of academic year 2011-2012

Students are able to draw lines of symmetry and to write the order of symmetry rotations. They are also able to understand the properties of angles, can name most of the shapes, understand reflection, rotation and translation .Besides they can differentiate between similar and congruent shapes. Moreover, they can examine, compare, and analyze properties of geometric figures. Line symmetry, predicting the results of sliding, flipping or turning two dimensional objects can also be understood and students are also able to investigate, describe and reason about combining and subdividing figures. Some challenging handouts can be given to students to practice these processes further.
The image in understanding a geometrical fact there are possible factors involved.
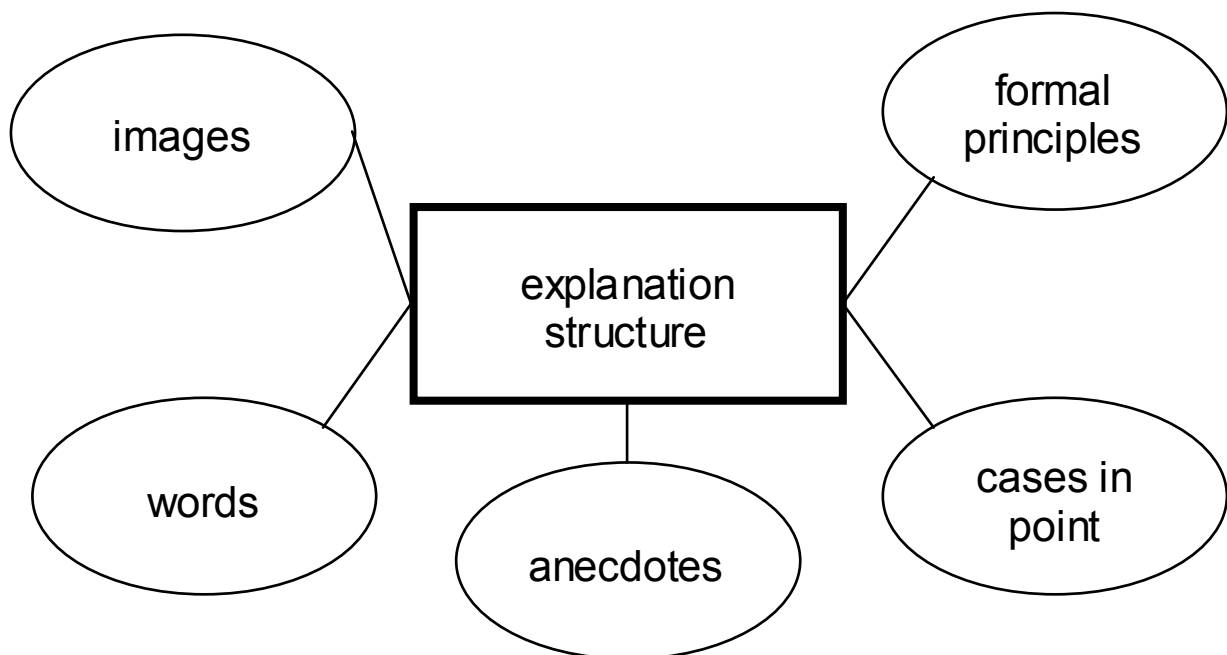

Fig.1: Possible factors involved in understanding a geometrical fact

In a student learning process there are some key factors such as network, images, words, anecdotes, cases in point , formal principles and finally explanation structures.

CONCLUSION

According to my research, it can be concluded that 7th grade students succeeded in reaching the curriculum objectives. My research aims to make the teachers aware of student misconceptions and general educational issues. The results from the study revealed that seventh year secondary school students have a number of misconceptions and lack of knowledge related to geometry subject.

Suggestions

The students couldn't remember the formulas after the exams because they just memorized them for the exams in short term memory not in long term memory. Some of the students were not able to keep the formulae in their long term memories because they could not create a positive attitude towards geometry and they were unable to associate it with their real lives. The cause of this problem can be related to the curriculum and teaching techniques used. The teachers may motivate students towards geometry. The formulae may be learned via applying and experiencing by the students.

The major problems in mathematics are inadequate thinking and reasoning abilities. The role of the teacher is very crucial to overcome this problem. The teacher should explain students what they should be careful about in image based questions in detail. In mathematics, teaching should be done in using visual aids. It was found out that students couldn't understand and evaluate mathematics, visual materials and methods which aim at students' five senses should be used to improve understanding . To succeed in geometry learning, it is very important to define objects and their definitions. Students get confused at recognizing the shapes. The reason for this is human perception. To eliminate this problem the teacher should first make students recognize the shapes then teach how to rotate the objects mentally to perceive them more clearly. According to the level of geometric thinking of the students, methods can vary.The teacher should continuously remind students that rotation of an object does not change its shape.

New practices for geometry lesson General suggestions

To teach students the names of various shapes, television, books and computer games can be used. Their comprehension of the concept should be improves. Their meta cognitive abilities should be enhanced.

- Teachers should use relevant vocabulary to describe relevant geometric statements and their relationships. To do this a teacher can not only identify assumption, hypothesis for geometric statements but also explain and show the role of definitions, conjectures, theorems, proofs and counter examples in mathematical reasoning by using geometric examples to illustrate these concepts .
- To assess the validity of geometric arguments a teacher should apply logic. A way of doing this is, analyzing the consequences of using alternative definitions for geometric objects.
- To help students memorize the formulas easier, the formulas can be shown with either proofs using different approaches.
- Simple geometric constructions should be analyzed, executed and applied.

The properties of geometric figures and mathematical thinking should be applied in order to perform and justify basic geometric constructions. Simple straightedge and compass constructions should be performed and explained In order to increase efficiency and reach aims computer based , visual methods are necessary.To test or create the conjectures of geometric properties or relations geometric computer or calculator packages can be used.

- Geometry sketchpad is a software that can be used for constructing basic geometric figures. It also enables you to edit and with the display menu you can add figures and animate them. It's custom tools let you replay complex geometric constructions in an easy one step way.
- Scheme of work : A detailed scheme which topics and what order topics should be covered.
- Practice book: They provide students plenty o exercises based on the content of the units.
- Case studies
- Optional substantial sections of work to cover much of the content of the modules. They aim to motivate the students so that they can get confidence needed in conjunction with the regular practice found in their practice books.
- Powerpoint Representations

For the upcoming years the curriculum of geometry can be revised by taking these factors into consideration:

Geometry curriculums have recently been reviewed. Geometry is both one of the oldest branches of mathematics and oldest form if intellectual achievement. It has been discussed what kind of purpose it should serve.

Following objectives should be taken into consideration to recommend to choose a geometry curriculum :

- Spatial awareness development, geometric intuition and visualization ability.
- Providing experiences in two and three dimensionals.
- Knowledge and understanding development to use geometrical properties and theorems
- Encouragement use of conjecture, deductive reasoning and proof development
- Real world contexts of developing skills of applying geometry through problem solving and modeling.
- Information and communication technology (ICT) skills develop useful geometrical contexts
- Developing a positive attitude to mathematics, an awareness to the contemporary applications of geometry and the historical and cultural heritages of it.

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