IJTASE

ISSN: 2146 - 9466
www.ijtase.net
International Journal of New Trends in Arts, Sports & Science Education – 2021, volume 10, issue 4

INVESTIGATION OF MISTAKES AND MISCONCEPTIONS OF 8TH GRADE STUDENTS ACCORDING TO STUMP'S SLOPE PERCEPTION CLASSIFICATION*

Abdulkadir ÖRNEKÇİ
Mathematics Teacher, Republic of Turkey Ministry of National Education,
Toplu Konut Secondary School, Palandöken, Erzurum, Turkey
ORCID: https://orcid.org/ 0000-0002-9815-4360
kadirornekci25@gmail.com

Ömer Faruk ÇETİN
Assoc. Prof. Dr., Erzincan Binali Yıldırım University, Faculty of Education,
Department of Mathematics and Science Education, Erzincan,Turkey
ORCID: https://orcid.org/0000-0002-3758-8747
ofaruk@erzincan.edu.tr

Received: 27-01-2021 **Accepted:** 01-04-2021 **Published:** 30-07-2021

Suggested Citation:
Örnekçi, A., & Çetin, Ö. F. (2021). Investigation of mistakes and misconceptions of 8th grade students according to stump's slope perception classification. *International Journal of New Trends in Arts, Sports & Science Education (IJTASE)*, *10*(4), 212-230.

Abstract
The aim of this research is to investigate eighth graders' mistakes and misconceptions of the "Slope" concept that take place in the mathematics curriculum and to make recommendations for solution. According to this purpose, the sample of the research consisted of 165 eighth graders from 4 different public schools located in Palandöken district of Erzurum province. The students of the sample were those, who were ready at the school on the day at which the research was carried out. To determine the mistakes and misconceptions of the students on the slope, a "Slope Test" consisting of 7 multiple choice questions and 8 open-ended questions, was created in accordance with the gains. Expert opinions were applied during developing the Slope test, and validity and reliance studies were carried out through a pilot study. In the research, the "Slope Test" was applied to the students and this test was used as the data collection tool. The descriptive scanning method was employed in the research and as the interview was held with some of the students if necessary, the mixed method was the method of the study. The obtained data were analyzed with the descriptive analysis and content analysis techniques. The mistakes and misconceptions of the students were examined according to the results of the study and some recommendations were made considering the results.
Keywords: Slope, misconception, eighth grade students, mathematics education.

INTRODUCTION

It is known that the concept of slope, which is expressed in various ways in our daily life such as a ramp, incline, gradient, enters the life of students from the pre-school period (Crawford & Scott, 2000). These cognitive situations, emerging with experiences, are still informal level and become formal first at the eighth-grade level of secondary school (Hoffman, 2015).

The teaching of the conceptual and operational integrity of these slope images (Styers, Nagle, & Moore-Russo, 2020). Acquired by students from daily life during the school term is significant for the students to fully learn the high-level concepts which are related to the slope they will encounter in the following years. However, as the teaching of the slope concept in schools remains at an operational level (Crawford & Scott, 2000), it is emphasized in the literature that there are difficulties in learning the concept of derivatives during which it is primarily necessary to know the slope (Barr, 1981; Clement, 1985; Cheng, 2010) and how to use the slope in solving problems in the following years (Zandieh, 2000)

* This article was produced from the first author's master's thesis titled "Examining the errors and misconceptions of eighth grade students about the subject of mathematics course education" and which was made under the supervision of the second author.

According to Barr (1981), the difficulties encountered related to the slope are listed below.

• The slope to y= 3x+2 equation is 3, but is 3 a ratio?

• Is the slope of the line, whose two points are known, the division of the difference of "x" to the difference of "y" or vice versa?

• Is the slope m or c in the equation y= mx + c?

• Inability to find the slope with two known points.

• Inability to find the slope when given a function that is the equation of two points and a curve.

One of the most significant reasons of why the concept of the slope cannot be taught at the desired level is due to the inability to comprehend the slope as the rate of change (Barr, 1981).

Crawford and Scott (2000) suggest that students should learn by experiencing the relationship between slope and rate of change by tabulating the problems involving real-life situations, graphing the obtained pairs in order to comprehend slope as the rate of changes.

Students, who learn the slope at the operational level, often focus only on the elevation and do not pay attention to the horizontal length (Clement, 1985; Duncan & Chick, 2013). In one example that supports these results, the students, who answered the question ‘among the saplings, whose length are different at the time they are planted and assumed to grown linearly, after which month does the short one pass the taller sapling?’ as ‘how the short one will pass the long saddling?’ only considered the growth amount during reading the elongation versus time graph; they said that the shorter sapling could never catch and pass the tall one as they did not determine the elongation rate by using the ratio of the change in height to the elapsed time. This example shows that students need to make sense of this concept both operationally and conceptually to use the slope in problem-solving.

According to Stump (1999), students interpreted the slope under 7 headings as age, level of education, etc. These are:

• Geometric ratio: The slope is the ratio of the vertical length to the horizontal length.

• Algebraic ratio: It is the ratio of the change in the vertical components of any two points on the line to the change in the horizontal components.

• Physical feature: The slope is the expressions of the ramp, incline, and gradient.

• Functional feature: It is the ratio of changes in two variables.

• Parametric coefficient: The slope of the line given equation in the form of y= ax+b is "a", which is the coefficient of variable x.

• Trigonometrically: It is the tangent of the positive directional angle of the line with the horizontal.

• Calculus (Analysis course) concept: Because of its relevance to derivatives, the slope is also a concept of calculus.

The individual's ability to fully explain perpendicularity measurement and his/her perception of the slope concept as a ratio is directly related to her/his success in linear algebra (Duncan & Chick, 2013). The prerequisite for the concept of derivative to be abstracted in the following years (Zandieh, 2000) eighth-grade students’ making sense of it at the conceptual level as well as the operative level during teaching the concept of slope and emphasizing the direction that slope is the rate of changes (Barr, 1981). In this context, students must learn the slope at the conceptual level so that they can solve mathematical problems including the slope and learn more about advanced mathematics, such as derivatives. For this, it is of great significance for students to start their high school education without any misconceptions about the slope.

Individuals can make mistakes in a variety of problem situations. What is important is where this mistake originates. If the individual is not aware of his/her mistake and insists on her/ his mistake, it is considered as the misconception (Yenilmez & Yaşa, 2008).

One of the reasons for the misconceptions is the previous life of the individual regarding the concept. Difficulties in transferring the informal knowledge to a formal level can create misconception (Invincible & Age, 2008). In addition, activities implemented in the school environment can produce misconceptions. In this regard, teachers need to design the learning environment in such a way that it does not produce the misconceptions. This may be possible by knowing and predicting the misconceptions that may occur among students on the subject (Özbellek, 2003). Therefore, it is useful to know the misconceptions that occur among students. In this context, in this study, the mistakes and misconceptions of students about the slope were tried to be revealed. The trigonometric aspect of the slope has not been explained with the primary mathematics course curriculum, which has been implemented since 2013 (MoNE, 2017). In addition, as the relationship of the slope with the derivative is included in the curriculum during high school years, 5 meanings and real-life situations from the 7 slope perception classification by Stump are taught to the students during the eighth class of the secondary school. Stump (1999)'s study will be taken as the basis of this study. In addition, students' mistakes and misconceptions about the gains in the eighth-grade mathematics curriculum will be investigated.

The aim of this research is to investigate the mistakes and misconceptions about the slope of eighth graders in a mathematics course and to make suggestions related to the issue.

Stump's (1999) study related to how students interpret slope according to age, level of learning, etc. and the study conducted by Barr (1981) on the difficulties encountered related to the slope.

The research problem of this study is "What are the mistakes and misconceptions of the eighth graders related to the slope according to Stump's slope perception classification?"

The sub-problems of the research are listed below.

1. Do the students make mistakes related to the slope? Do they have misconceptions?
2. What are the mistakes and misconceptions according to Stump's Slope perception classification?

METHODS

Research Design

The model of the research is the mixed method as the data were obtained through the descriptive scanning method and interviews were made with some students when necessary. Creswell and Clark (2007) describe the mixed method research as the research method that enables data collection, analysis, and integration with the qualitative and quantitative methods. According to the mixed method, each case and phenomenon has both qualitative and quantitative dimensions. If it is necessary to be understood as the real, qualitative, and rich framework, both qualitative and quantitative dimensions of it should be examined. As the mistakes and misconception of the students were investigated in this study, it would be significant to examine the incorrect answers of the students in order to determine whether the incorrect answers to the questions were a mistake or a misconception and to understand whether the students insisted on their mistakes. Therefore, the mixed method in which quantitative and qualitative methods are applied together was used in this study. It is stated that mixed-method research has five important functions. These functions are listed as variation, completion, development, startup, and expansion. These functions are basically based on the following: "Data obtained from qualitative (or quantitative) methods are used to confirm or improve data obtained by other methods. It is an important goal to investigate the same dimensions of a research problem together with both methods and to ensure data diversity and confirmation in this way (Greene et al., 1989). In this study among the mixed-method research patterns, the triangulation pattern was used. The purpose of the triangulation pattern is to use qualitative and quantitative data together, to diversify, compare, intensify the data obtained, and to obtain data directly related to different but research questions (Morse, 1991). Qualitative and quantitative data can be used for various purposes when using triangulation patterns (Yıldırım & Şimşek, 2016). In this study, qualitative data was used for the analysis of quantitative data. In this context, in cases at which the students' answers to questions or explanations are not understood, the minutes of the semi-structured interview with them are arranged and they are asked to provide more detailed information about their solutions or explanations.

Participants

The universe of the research consisted of eighth-grade students who had already learned the slope subject. In this context, the sample of the research consisted of the students, who were at the eighth grade of 4 different state schools with moderate achievement level in the central Palandöken district of Erzurum province in the 2016-2017 academic year. In determining the sample of the research, no selection process was performed, among all the students, the study was carried out with 165 students who came to the school on the day of the slope test was applied.

Data Collection Tools

The data of the study were obtained with the "Slope Test" applied to all 165 students consisting of the sample and the "Interview Reports" applied to the students deemed necessary. Below is information about the "slope test" and "interview reports".

Slope Test

The literature was reviewed during preparing the slope test and questions were developed considering the students' understanding of the slope as well as the difficulties related to the slope. In this context, the slope test prepared in accordance with the two-stage diagnostic tests is a multiple-choice including 4 options that will reveal the students' misconceptions that can be revealed in the first 7 questions, and the other 8 questions are open-ended questions that require solutions and explanations. Open-ended questions in the second section of the test, there are questions that control multiple choice questions in the first section. These questions were not asked one after another but were distributed into the test a mixed way. For example, question 1 and question 12 are chosen to assess the same gain. The researcher's 14 years of teaching experience was beneficial in preparing the options. The relevant questions, the skills measured by the questions, and what meaning of slope these questions are related to according to Stump (1999) are presented in Table 1.

Table 1. Relevant questions, the knowledge/skills measured by the questions and meanings.

Relevant Questions	Measured Knowledge/Skill	Skill
1st and 12th Questions	Telling the slope is the ratio of the vertical length to horizontal length.	Geometric meaning
2nd and 13th Questions	Telling that the perpendicular angle of the line with the horizontal is the slope angle of that line	Trigonometric meaning
3rd and 10th Questions	Finding the slope in real life situations	Real life
4th and 9th questions	Finding the slope of the line whose equation is given	Parametric coefficient
5th and 14th Questions	Finding the equation of the line whose slope and on point has already known	Algebraic ratio and parametric coefficient
7th and 8th Questions	Finding the slope of the lines parallel to the axis Finding the slope of the line with known two points	Geometric ratio and algebraic ratio
6th and 11th Questions	Finding the slope of a line whose graph has already given	Geometric ratio
15th Question	Finding the slope in real life situations	Physical meaning and real life status

The 7th and 8th questions, presented in the table above having correlation, questioning the meaning of the slope's geometric ratio and algebraic ratio, are presented in Figure 1.

7. QUESTION: What is the slope of the right given by the equation y=4?

a) 1 b)4 c) 0 (Zero) d) No slope (indefinite)

8. Find the slope of the right passing through points A(5,3) and B(7.3). Briefly explain how you found it.

Figure 1. 7th and 8th questions relevant to each other.

Both questions given in Figure 1 question the geometric ratio and algebraic ratio meanings of the slope. In addition, question 8 with an explanation controls question 7 which is in multiple-choice type.

After the slope test was prepared, a pilot study was held with 25 eighth graders who were studying at a state school and necessary adjustments were given to the question roots misunderstood by the students. Final form was given to the test after the opinions of other mathematics teachers at the school and a field expert.

Interview reports

In a quiet classroom environment, students were reminded of some of the questions they answered and their answers to these questions, an opportunity was given them to re-examine their papers, and asked to state some more information about their explanation. The students' answers to multiple choice questions and their answers to questions requiring clarified questions were compared and the students to be interviewed were determined as a result of this comparison. For example, if a student has answered the multiple-choice question correctly and answered the question correctly, if he or she has made an incorrect statement, this student was selected to be interviewed. They are also reminded that their answers to the test will not affect their grades in any way.

Data Collection Process

The 165 eighth graders, to whom the subject of slope was previously taught, were applied with the “Slope Test” in the classroom environment, in mathematics course and under the supervision of their mathematics teacher. The test lasted in 40 minutes. Semi-structured 5-10-minute interviews were held with students who did not fully understand the answer to the question requiring explanation. In these interviews, by asking "You answered that question, and you explained the solution, can you give more explanation what you mean here"? to understand what they actually mean. Interviews were reported in documents.

Data Analysis

In the analysis of the data, we looked at whether the students answered the multiple-choice questions correctly, which is the first part of the slope test, organized in the form of a two-stage diagnostic test. Each question in the first part of the test and the relevant question in the second phase were evaluated together. For example, we gave attention whether the students, who answered question 1 incorrectly, answered the relevant question, question 12, incorrectly, or not. In this process of the data analysis, students' answers to questions were analyzed with the content analysis. Yıldırım and Şimşek (2016) defined the main purpose of content analysis as "identifying data and uncovering facts that may be hidden within the data". Within the framework of content analysis, data are analyzed in four stages: the 1st is the determination of codes, the 2nd is creation of categories, the 3rd is the regulation of codes and categories, and the 4th is identification and interpretation of findings (Yıldırım & Şimşek, 2016).

Analysis of questions in the first stage was performed as; if the answer is correct, the code is "True", and if the code is incorrect or not answered, the code is "Empty".

The analysis of the answers to the questions in the second phase of the test and the data gathered from the explanations were performed as; if the answer and explanation is correct, the code is "True", if the answer and the description is incorrect, the code is "Incorrect", if the answer is incorrect- if there is no explanation, if the answer is "Incorrect", if the answer is true and there is no explanation, the code is

"Partially Incorrect", if the answer is incorrect-explanation is correct, the code is "Partially Incorrect" and if there is no answer, the analysis is performed according to the code of "Empty".

The items in the first stage of the test were multiple choice and the following roadmap was used to determine whether students had a misconception by considering the codes obtained and that answering the questions at this stage incorrectly or correctly would not be enough to make judgments such as "there is/is not misconception" alone (as the answer can be correct by random marking or incorrect as a result of a minor operation error). Accordingly, during the analysis of the data, the situations as,

• Giving “incorrect” answer to the relevant questions in both parts of the test.

•Leaving the question in the first part of the test “empty” and answering the relevant question in the second part "incorrectly", were taken as a sign of misconceptions, and these situations were presented with the category "there is a misconception".

The situations,

• Giving “correct” answer to the questions in both parts of the test,

• Leaving the question in the first part of the test “empty” and answering the relevant question in the second part "correctly", were presented with the category "answering correctly".

The situations as,

• Giving "correct", "incorrect" and “empty” answer the question in the first part of the test and answering the relevant question in the second part "partially incorrect", were presented in the category "making mistakes".

The situations as

• Answering the question "incorrect" in the first part of the test and answering the relevant question in the second part "correctly",

• Answering the question in the first stage of the test "correct", and answering the relevant question in the second stage "incorrect", were regarded as a sign that these students should be interviewed and categorized as "answering correctly", "has a misconception", "making mistakes" and "no results" as a result of the interview.

The 15th question in the test consists of two parts in itself. In both parts, the reflection of the slope in real-life situations was analyzed and categorized as following.

• Considering the direction of the lines, calculating the slope and the status of emphasizing this answer in the statement were presented with the category of "having misconception",

• The region with a large slope due to its vertical/horizontal ratio is zone 2; the ball will be forced is also the 4th zone because the ramp is up situations were presented with the category "answering correctly",

• The sentences of "The slope of the 2nd region is the largest. As the slope is the largest, it is forced in zone 4" or the description status with close understanding with these sentences were presented in the category of "making mistakes".

RESULTS

In this section, the findings obtained with the analysis of the answers of the students to the relevant questions are presented.

Findings Related to the First Sub-Problem

In this section, the findings related to the sub-problem "Do the students make mistakes about slope? Are there any misconceptions about the concept?" are presented.

Among the students' answers to the related questions, the mistakes, and misconceptions about slope in terms of frequency and percentage according to Stump's seven classification, are presented in Table 2 below.

As it is understood from the data in Table 2, there are several students who make mistakes for all meanings of the slope and have a misconception. For example, 19 students made mistakes in the questions, in which they explain the slope as the "geometric meaning" and totally 20 students had misconceptions.

Table 2. The mistake and misconception frequency and percentage values related to slope of the from students' answers to the relevant questions and 15th question.

Relevant Questions	Meaning	Code Obtained in The First Stage						Code Obtained in Part Two	Category
		Correctly		Incorrect		Empty			
		f	%	f	%	f	%		
1 and 12	Geometric meaning	15	9.10	4	2,4	0	0	Partially incorrect	Making mistake.
		10	6.06	10	6,06	0	0	İncorrect	Having misconception
2 and 13	Trigonometric meaning	19	11.50	16	9.70	0	0	Partially incorrect	Making mistake.
		12	7.30	27	16.40	0	0	Incorrect	Having misconception
3 and 10	Real life status	3	1.80	6	3.60			Partially incorrect	Making mistake.
		13	7.90	22	13.30			Incorrect	Having misconception
4 and 9	Parametric coefficient	1	.60	3	1.80			Partially incorrect	Making mistake.
		17	10.30	12	7.30			Incorrect	Having misconception
5 and 14	Algebraic ratio and parametric coefficient	4	2.40	1	.60			Partially incorrect	Making mistake.
		10	6.70	13	7.90			Incorrect	Having misconception
7 and 8	Geometric ratio and algebraic ratio	13	7.90	11	6.70	1	.60	Partially incorrect	Making mistake.
		27	16.40	35	21.20	3	1.80	Incorrect	Having misconception
6 and 11	Geometric ratio	6	3.00	3	1.80			Partially incorrect	Making mistake.
		40	24.20	34	20.60			Incorrect	Having misconception
						f		%	
Question 15	Physical meaning and real-life status					31		18.80	Making mistake
						30		18.20	Having misconception
						89		54	Answering correctly.
						13		7.90	No solution

As it can be seen from Table 2, 31 students made mistakes for the "Physical meaning and real life" status of the slope from the data obtained from the answers to question 15 and 30 students have misconceptions.

Mistakes and misconceptions related to the slope from students' answers to the relevant questions are presented in Table 3 below according to Stump's seven classifications.

Table 3. Mistakes and misconceptions about Slope from students' answers to the relevant questions.

Meaning	Mistakes	Concept Misconception
Geometric meaning	Description of horizontal length as slope without proportioning vertical length to horizontal length.	□ Finding the slope by proportioning the lengths of any given sides (regardless of whether they are vertical or horizontal), □ To express the slope as "ratio of horizontal length to vertical length", □ Expressing only the length of the perpendicular edge as a slope.
Trigonometric meaning	Explaining that the slope gets larger as the angle gets smaller	•Expressing the given angle as a slope angle without paying attention to the position of the angle given in the triangle. •Describing the angle with the maximum extent as a slope angle, •Defining the slope angle of a line as the angle at which it made to the vertical.
Real life status	Inability to sort fractions. Sometimes taking the direction of the line into consideration but sometimes not.	Considering when the direction of the line should not be taken into account (using expressions such as "the slope is negative because it is tilted to the left" in the explanations).
Parametric coefficient	Inability to determine the coefficient of variable "x" in the given line equation (algebraic error).	□Ignoring the coefficients of variables, □ Expressing that the constant term in the given equation is slope.
Algebraic ratio and parametric coefficient	Inability to determine a point in the coordinate plane	Inability to express the equation of the line given the slope and a point it passes through.
Geometric ratio and algebraic ratio	Inability to comprehend the set of rational numbers.	• Inability to find the slope of trues parallel to the axis (expressing y=4 correctly; "no slope" by ensuring its parallelism of this line to the "x" axis with the informal knowledge that "there is no slope", even though y=4 l line paints the correct one) •Taking a point such as A(x,y) as two points (x.0) and (0.y), •Inability to find the slope of the line with two points given
Geometric ratio	Lack of direction knowledge	When you find the slope of the given line in the coordinate plane, don't take into account the direction of the line, just taking the points where the line cuts the axis into consideration.
Physical meaning and real-life status	Inability to express the solution in writing	•Referring "If the slope is going uphill, the slope is large, if downhill, the slope is small", •Referring, "Being right-slanted requires having a large slope and being left-slanted requires a small slope."

There are some examples of possible causes and data for mistakes that are given for each meaning in Table 3 below.

The reason for the mistake for the "geometric meaning" of the slope may be because students are distracted when faced with multiple process stages within the same question, moving away from the question of what they are looking for. Students who made this mistake found the edge not given using Pythagorean relation in question 12 but could not find the slope.

For example, the answer of S16 to the 12th question is shown in Figure 2.

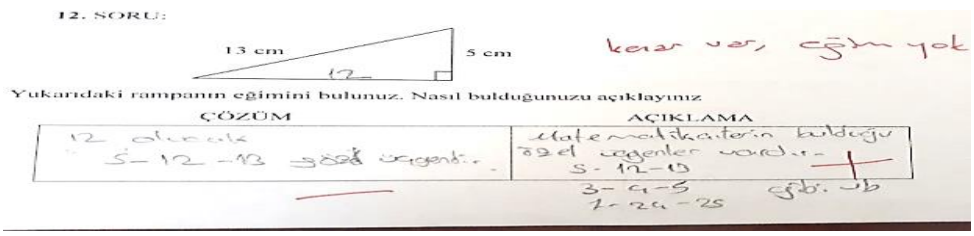


Figure 2. S16's answer to question 12

Although the error given for the meaning of "trigonometric" of the slope seems to be a misconception at first sight, as the students' solutions were correct, interviews were held with these students about their explanations and was found that they were all think in the right way but unable to express their opinions. Therefore, the situation under discussion was not considered a misconception, but as a mistake.

There are the interview report with S63, one of the students who explained that "the slope grows as the angle shrinks", are given in Figure 2.

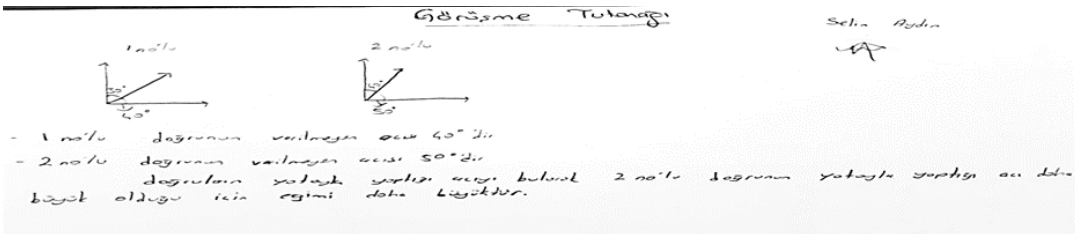


Figure 2. Interview minutes with Ö63

Although some of the students who made this mistake know the concept of "slope angle", it is seen that they use incorrect expressions (The smaller the angle, the greater the slope). Examples that can be given to this situation are the answers given by S82 in Figure 3 and Figure 4.

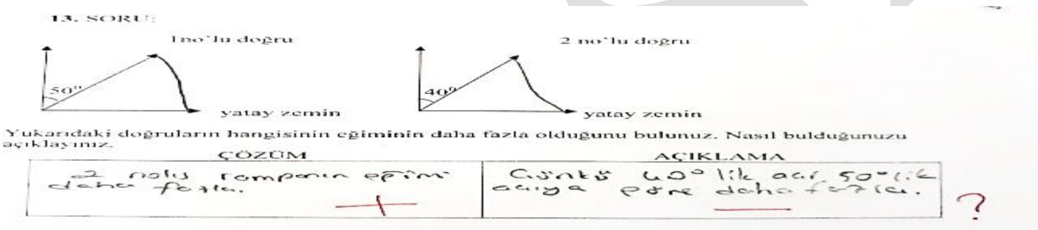


Figure 3. S82's answer to question 13

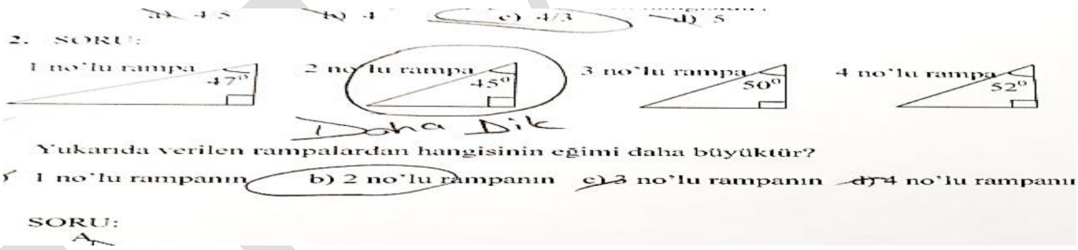


Figure 4. S82's answer to question 2

The answer to the S27 for the mistake in the "real-life" situations of the slope is presented as a example in Figure 5.

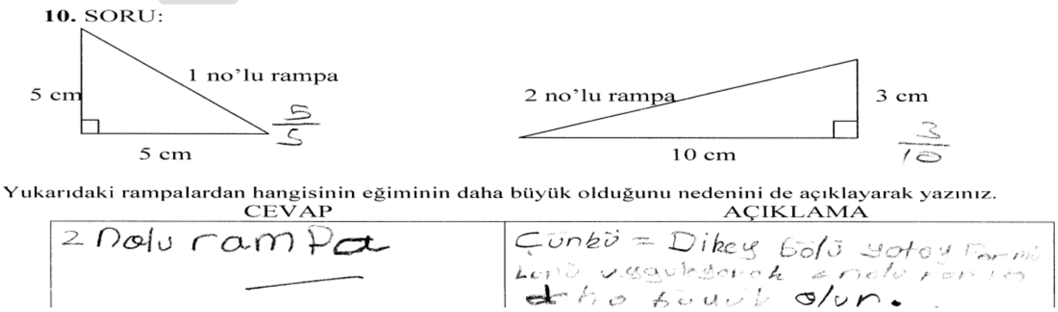


Figure 5. The answer of S27, which failed to sort fractions.

As can be seen in Figure 4, the student found the slope correct, but was not able to sort the fractions.

Another mistake with real-life situations has sometimes emerged as sometimes taking the direction of line into account and sometimes not.

For example, the answers, given by S116 from students who made mistake, are presented in Figure 6 and Figure 7.

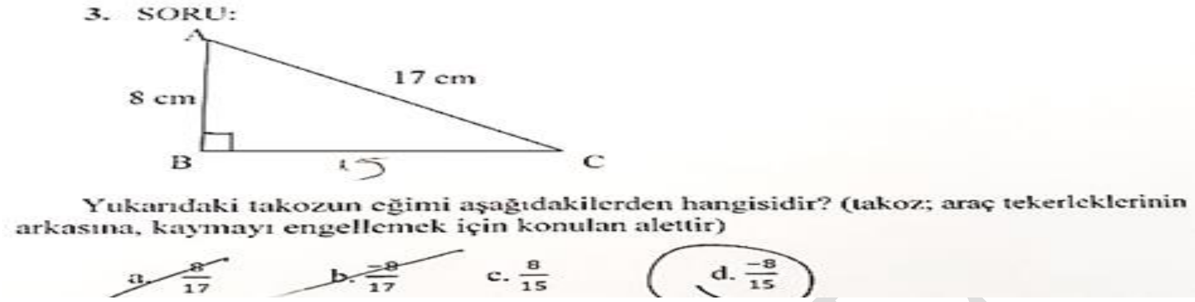


Figure 6. S116's answer to question 3

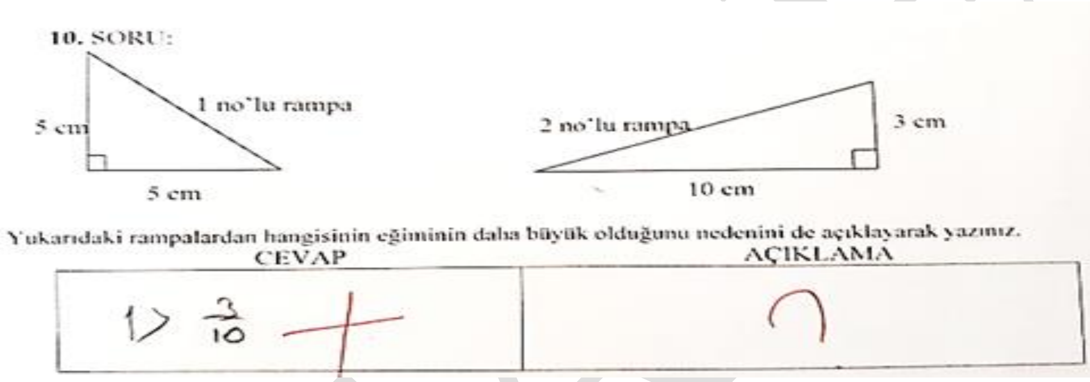


Figure 7. S116's answer to question 10

In spite of stating that "I pass the coefficient of y other side to leave y alone, and the coefficient in front of x becomes the slope", the reason for the mistake made for the meaning of the "parametric coefficient" of the slope is the coefficient of the y variable is taken as a slope in the equation without any influence on the coefficient of the "x" variable when it passes the coefficient of the y variable to the opposite side of equality, not as a section. Examples of this situation are the answers given by S50 in Figure 8 and Figure 9.

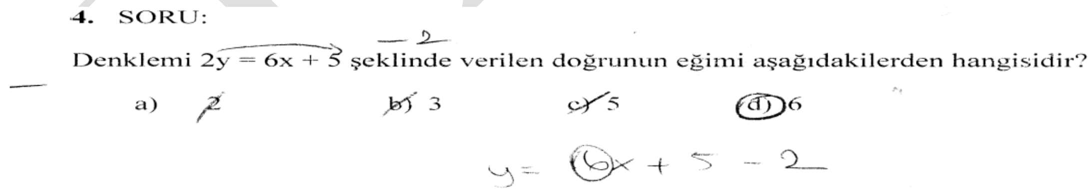
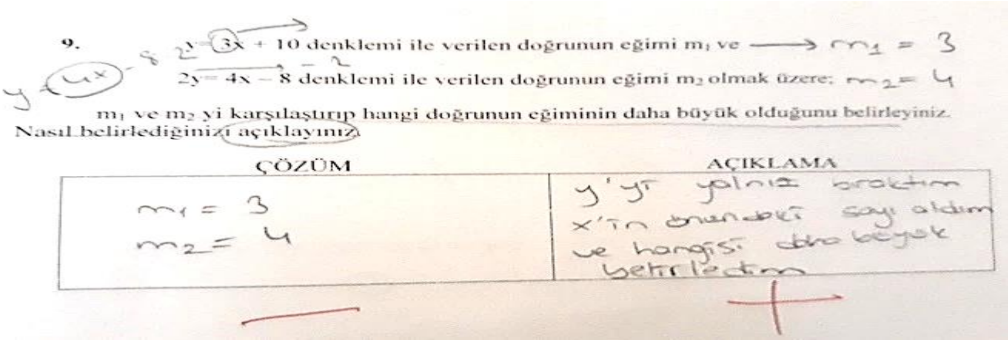


Figure 8. S50's answer to question 4

ISSN: 2146 - 9466
www.ijtase.net
International Journal of New Trends in Arts, Sports & Science Education – 2021, volume 10, issue 4


9. y = 3x + 10 denklemi ile verilen doğrunun eğimi m₁ ve 2y = 4x – 8 denklemi ile verilen doğrunun eğimi m₂ olmak üzere; m₁ ve m₂ yi karşılaştırıp hangi doğrunun eğiminin daha büyük olduğunu belirleyiniz. Nasıl belirlediğinizi açıklayınız.

ÇÖZÜM	AÇIKLAMA
$m_1 = 3$ $m_2 = 4$	y'yi yalnız bıraktım x'in önündeki sayıyı aldım ve hangisi daha büyük belirledim

Figure 9. S50's answer to question 9

The answers of S84 are presented in Figure 10 and Figure 11 for the example of the mistake related to the meanings of the slope as "algebraic ratio and parametric coefficient".

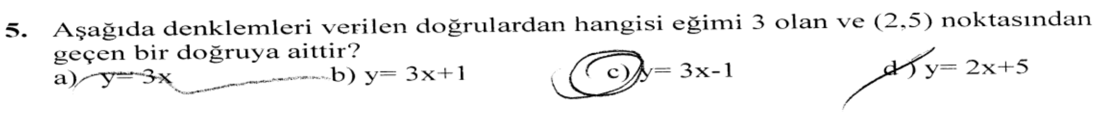
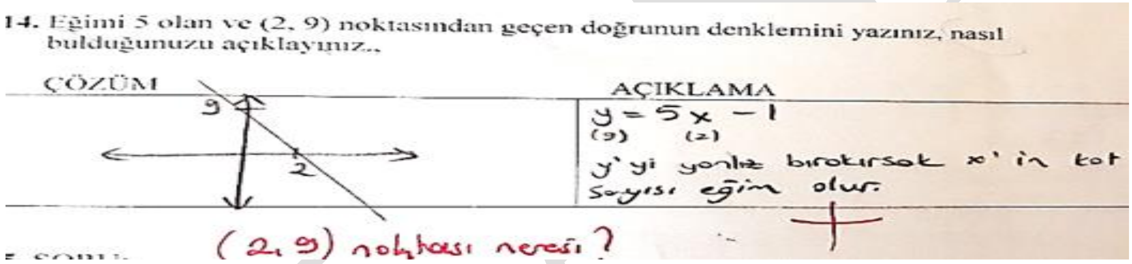

5. Aşağıda denklemleri verilen doğrulardan hangisi eğimi 3 olan ve (2,5) noktasından geçen bir doğruya aittir?
a) y= 3x b) y= 3x+1 c) y= 3x-1 d) y= 2x+5

Figure 10. Answer of S84 to question 5

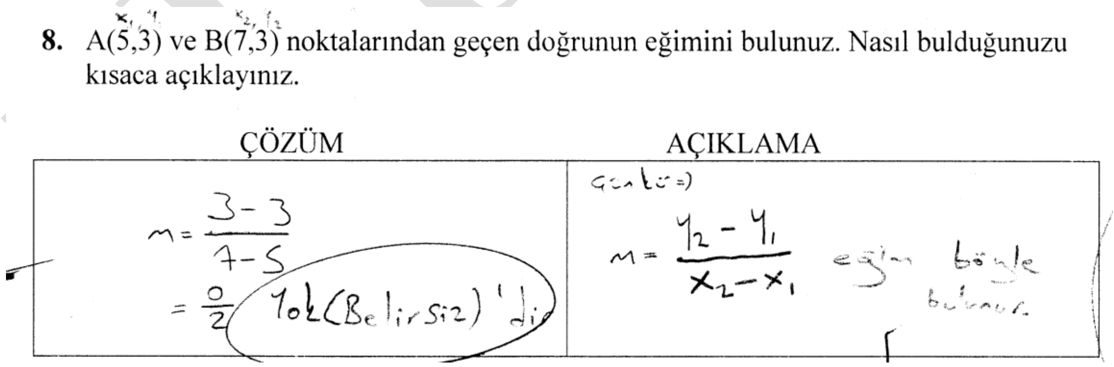

14. Eğimi 5 olan ve (2, 9) noktasından geçen doğrunun denklemini yazınız, nasıl bulduğunuzu açıklayınız..

ÇÖZÜM	AÇIKLAMA
	$y = 5x - 1$ y'yi yalnız bırakırsak x'in kat sayısı eğim olur.

(2, 9) noktası nerede?

Figure 11. Answer of S84 to the question 14

The solution to question 8 of S54, who made mistake for misinterpreting the result of the operation even though s/he knows the way to resolve the mistake for the meaning of "geometric ratio" and "algebraic ratio" is presented in Figure 12.


8. A(5,3) ve B(7,3) noktalarından geçen doğrunun eğimini bulunuz. Nasıl bulduğunuzu kısaca açıklayınız.

ÇÖZÜM	AÇIKLAMA
$m = \frac{3-3}{7-5} = \frac{0}{2}$ Yok (Belirsiz)'dir	$m = \frac{y_2 - y_1}{x_2 - x_1}$ eğim böyle bulunur.

Figure 12. Answer of S54 to question 8

The answer of S16 to the 11th question as an example of the mistake determined the meaning of the "geometric ratio" of the slope, is presented in Figure 13.

ISSN: 2146 - 9466
www.ijtase.net
International Journal of New Trends in Arts, Sports & Science Education – 2021, volume 10, issue 4

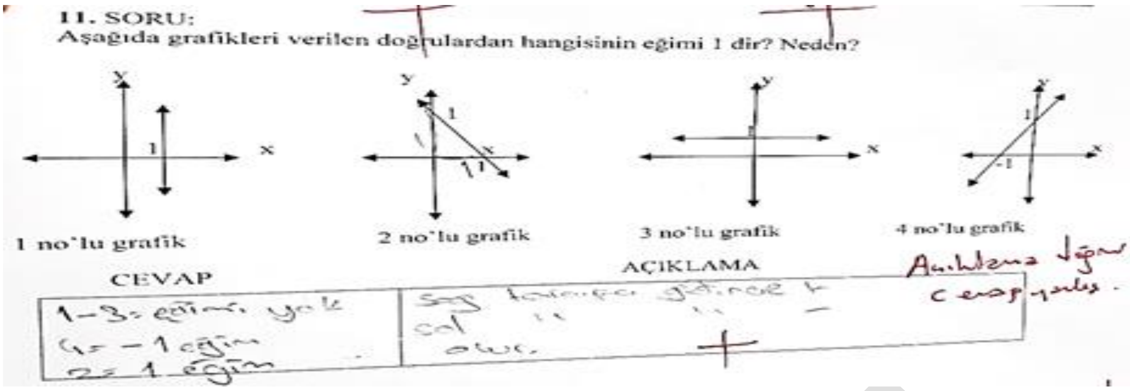


Figure 13. S16's answer to question 11

The answer to the 11th question of S146, who solved the question but could not express the solution in writing, is presented in Figure 14 as an example of the mistake made for the meaning of the "physical meaning and real-life status" of the slope.

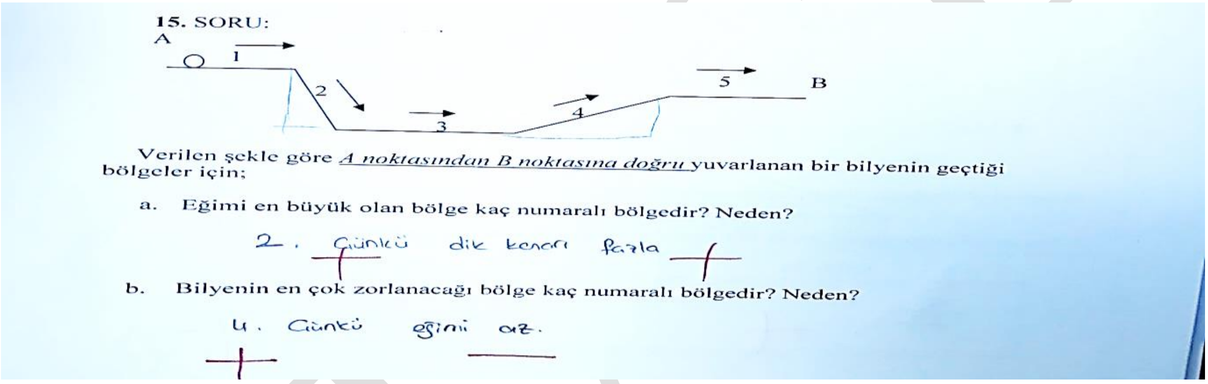


Figure 14. S146's answer to question 15

The answers of S37 to the 1st and 12th questions are given in Figure 15 and Figure 16. as an example of the misconception determined "geometric meaning" of the slope.

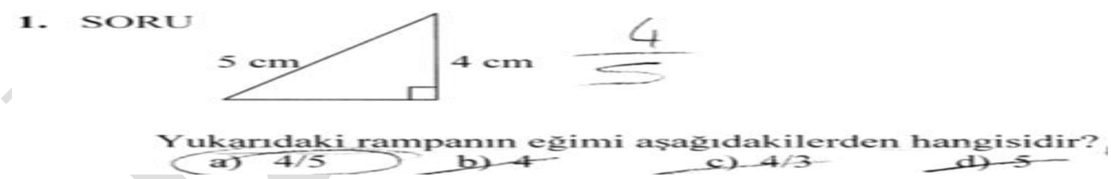


Figure 2. S37's answer to question 1

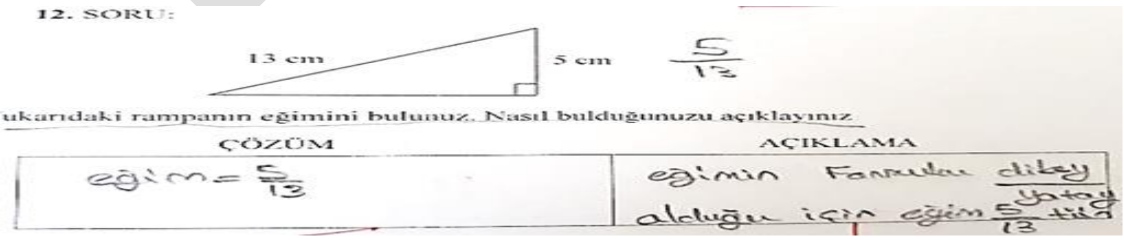


Figure 16. S37's answer to question 12

The answers of S104 are presented in Figure 17 and Figure 18 as an example of the misconception determined for "trigonometric" meaning of the slope.

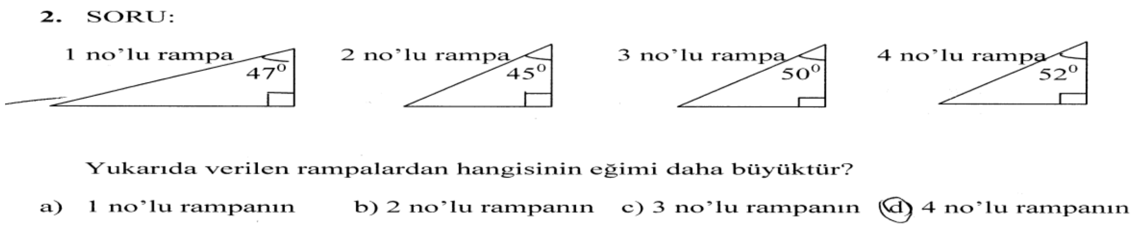


Figure 17. S104's answer to question 2

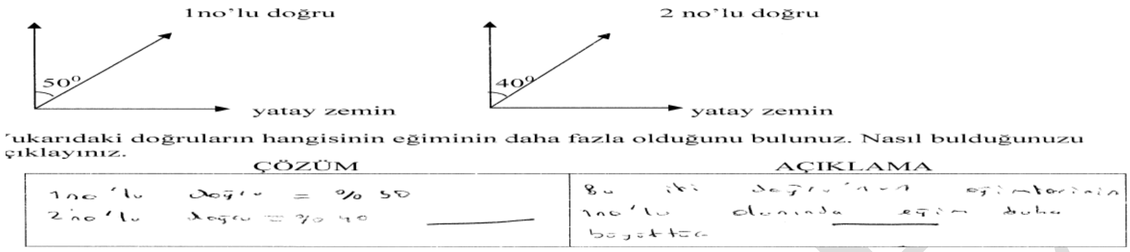


Figure 3. S104's answer to question 13

The answers given by S83 are presented in Figure 19 and Figure 20 as an example of the misconception determined for "real life" meaning of the slope.

Figure 19. S83's answer to question 3

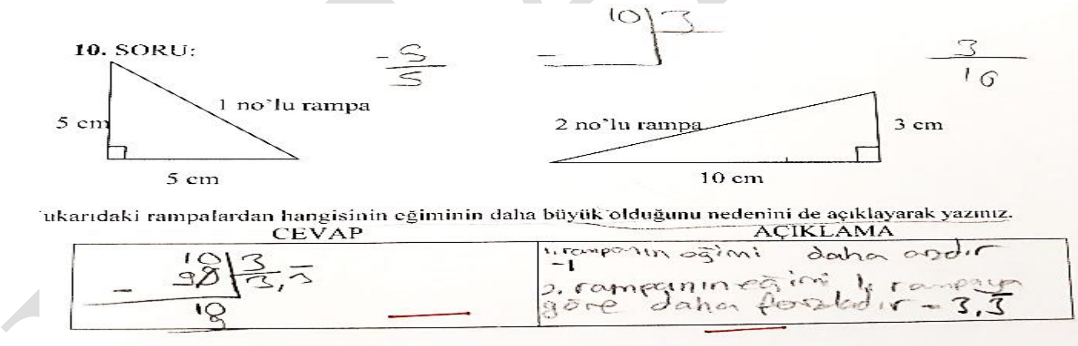


Figure 20. S83's answer to question 10

The answers of S63 are presented in Figure 21 and Figure 22 as examples of the misconception that determined as the "parametric coefficient" meaning of the slope.

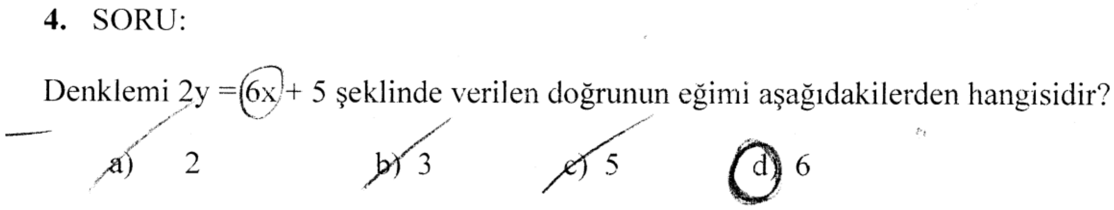
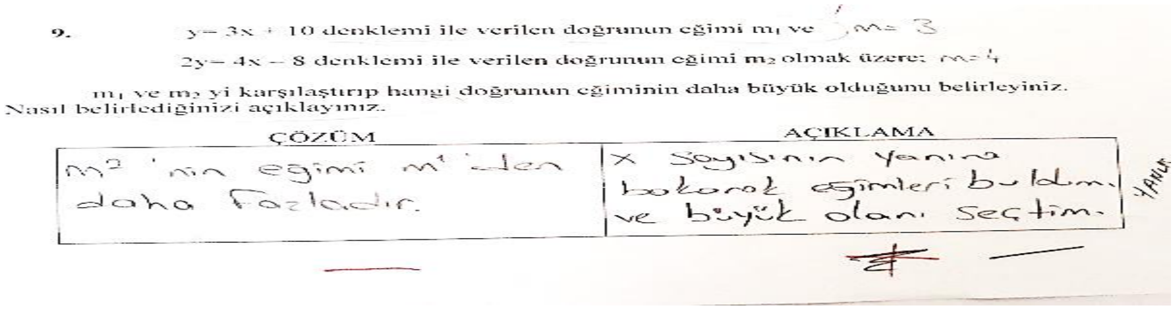


Figure 21. S63's answer to question 4

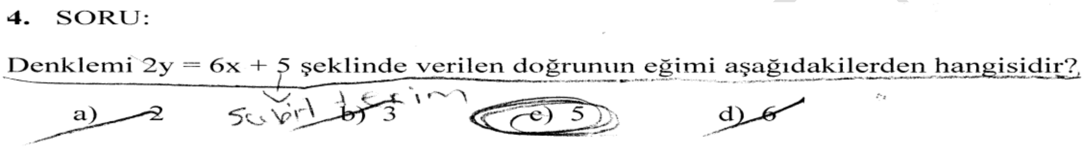


9. y= 3x + 10 denklemi ile verilen doğrunun eğimi m_1 ve ; m= 3
2y= 4x – 8 denklemi ile verilen doğrunun eğimi m_2 olmak üzere: m=4
m_1 ve m_2 yi karşılaştırıp hangi doğrunun eğiminin daha büyük olduğunu belirleyiniz.
Nasıl belirlediğinizi açıklayınız.

ÇÖZÜM	AÇIKLAMA
m² 'nin eğimi m¹ 'den daha fazladır.	x sayısının yanına bakarak eğimleri buldum ve büyük olanı seçtim.

Figure 22. S63's answer to question 9

In another example, the answer of S30 is presented in Figure 23 from the students who describe the line in the equation as a slope in finding the slope of the given line.



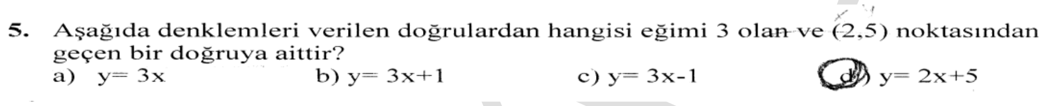
4. SORU:

Denklemi 2y = 6x + 5 şeklinde verilen doğrunun eğimi aşağıdakilerden hangisidir?

a) 2 sabit terim b) 3 c) 5 d) 6

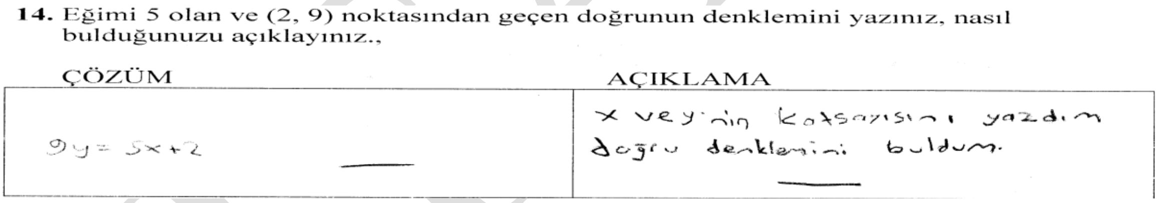
Figure 23. S30's answer to question 4

Examples of the misconception that the slope, determined for the "algebraic ratio and parametric coefficient" are presented in Figure 24 and Figure 25.



5. Aşağıda denklemleri verilen doğrulardan hangisi eğimi 3 olan ve (2,5) noktasından geçen bir doğruya aittir?
a) y= 3x b) y= 3x+1 c) y= 3x-1 d) y= 2x+5

Figure 4. S72's answer to question 5

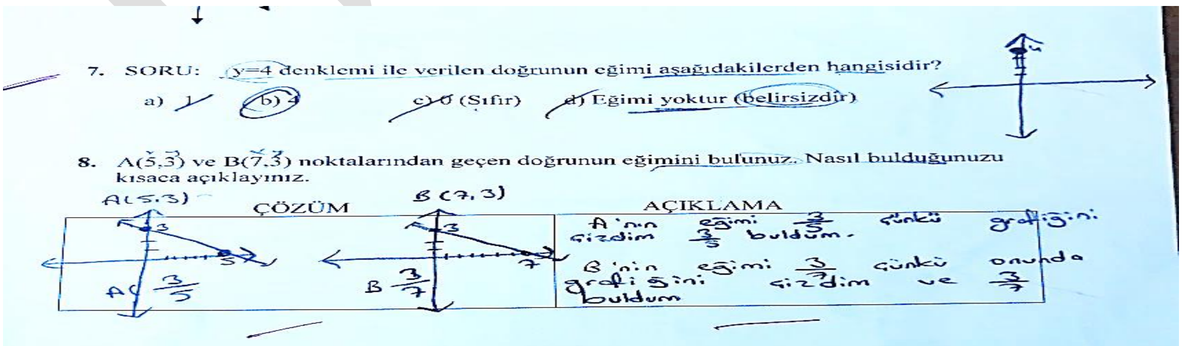


14. Eğimi 5 olan ve (2, 9) noktasından geçen doğrunun denklemini yazınız, nasıl bulduğunuzu açıklayınız.,

ÇÖZÜM	AÇIKLAMA
9y= 5x+2	x ve y'nin katsayısını yazdım doğru denklemini buldum.

Figure 25. S72's answer to question 14

The answers of S72 given as the examples of the misconception that the slope is determined for the "algebraic ratio and parametric coefficient" are presented in Figure 26.



7. SORU: y=4 denklemi ile verilen doğrunun eğimi aşağıdakilerden hangisidir?
a) 1 b) 4 c) 0 (Sıfır) d) Eğimi yoktur (belirsizdir)

8. A(5,3) ve B(7,3) noktalarından geçen doğrunun eğimini bulunuz. Nasıl bulduğunuzu kısaca açıklayınız.

ÇÖZÜM	AÇIKLAMA
A(5,3) A $\frac{3}{5}$ B(7,3) B $\frac{3}{7}$	A'nın eğimi $\frac{3}{5}$ çünkü grafiğini çizdim $\frac{3}{5}$ buldum. B'nin eğimi $\frac{3}{7}$ çünkü onunda grafiğini çizdim ve $\frac{3}{7}$ buldum

Figure 26. S1's answers to questions 7th and 8th

The answer of S40, who found the slope of the line, with to two points given, incorrect, is presented in Figure 27.

ISSN: 2146 - 9466
www.ijtase.net
International Journal of New Trends in Arts, Sports & Science Education – 2021, volume 10, issue 4

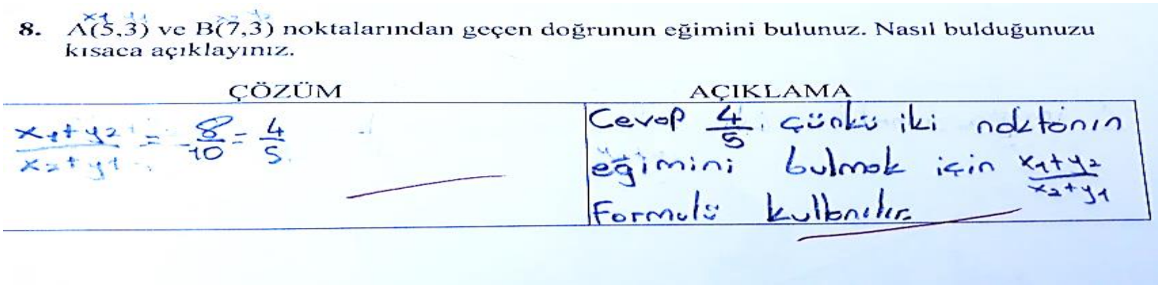


Figure 27. S40's answer to question 8

The answers of S36 as the examples of the misconception determined for the "geometric ratio" of the slope are presented in Figure 28 and Figure 29.

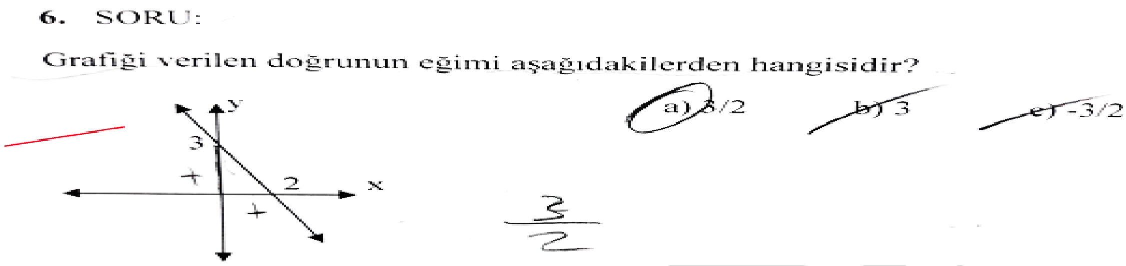


Figure 28. S36's answer to question 6

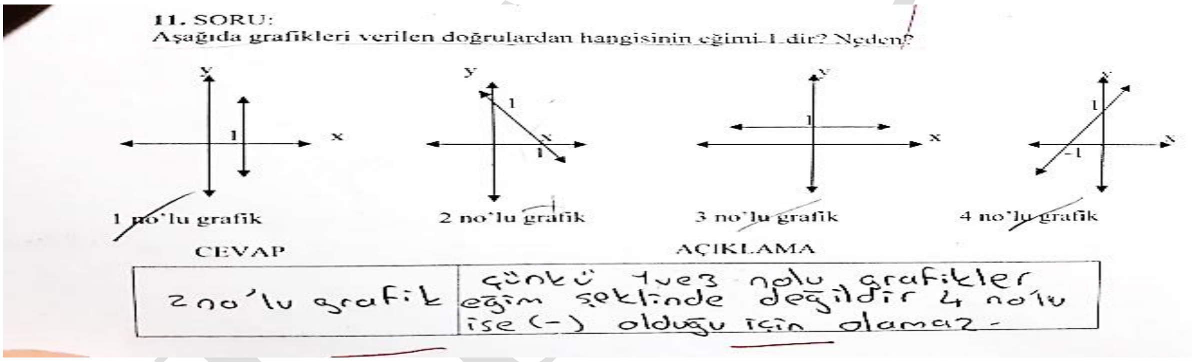


Figure 29. S36's answer to question 11

The answers of S98 and S105 as examples of misconception, determined for "physical meaning and real-life status" of the slope, are presented in Figure 30 and Figure 31.

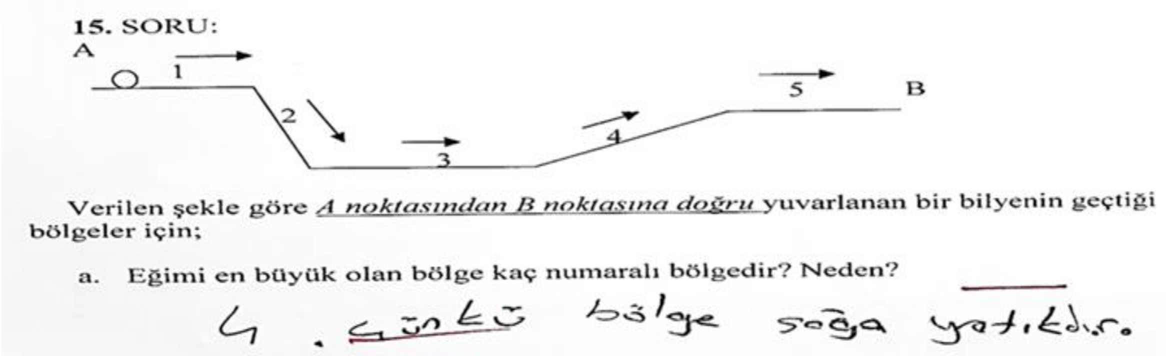


Figure 30. S98's answer to question 15

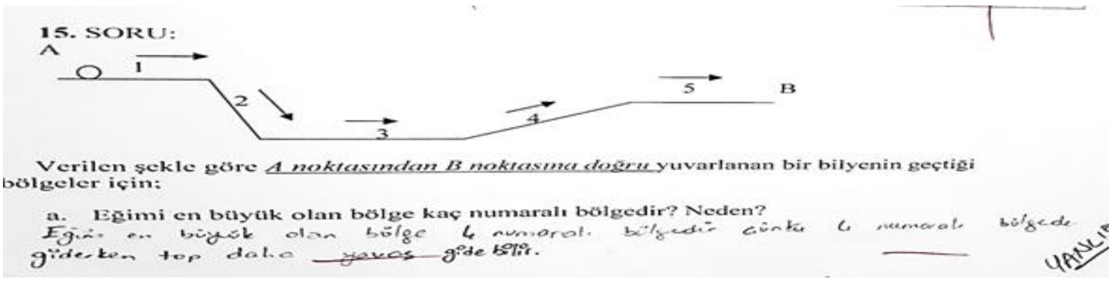


Figure 31. S105's answer to question 15

Findings Related to the Second Sub-Problem

In this section, findings Related to the sub-problem "What are the mistakes and misconceptions according to Stump's Slope perception classification?" will be presented.

The findings that the students explained the slope within the framework of the 7 areas Stump (1999) are presented in Table 3 below.

Table 3. Findings of the students' explaining the slope within the framework of the 7 areas Stump (1999)

Question(s)	Meaning	Mistake Rate (%)	Misconception Rate (%)	Correct Rate (%)	No solution Rate (%)
1 and 12	Geometric meaning	11.5	12.1	67.3	9.1
2 and 13	Trigonometric meaning	21.2	23.6	39.4	15.8
3 and 10	Real life status	5.5	21.2	69.7	3.6
4 and 9	Parametric coefficient	2.4	17.6	56.4	23.6
5 and 14	Algebraic ratio and parametric coefficient	3	13.9	26.1	57
7 and 8	Geometric ratio and algebraic ratio	15.2	39.4	21.8	23.6
6 and 11	Geometric ratio	4.8	44.8	47.3	3
15	Physical meaning and real-life status	18.8	18.2	55.2	7.9

As it is understood from the Table 3, 67.3% of the students gave correct answers for the geometric meaning of the slope. For the meaning of the trigonometric ratio of the slope, it is understood that 39.4% of students gave correct answers, 21.2% incorrect, and 23.6% had misconceptions. For the real-life situation of the slope, it is observed that 69.7% of students gave the correct answer and 21.2 % had misconceptions. For the parametric coefficient meaning of the slope, it is observed that 56.4% of students gave correct answers, 17.6% had misconceptions, and only 2.4% of students made mistakes. It is observed that 57% of students did not answer the questions related to the meaning of the algebraic ratio and parametric coefficient of the slope. It is observed that 39.4% of the students had various misconceptions for the meaning of geometric ratio and the algebraic ratio of the slope. For the geometric ratio meaning of the slope, it is observed that 47.3% of the students gave correct answers and 44.8% had misconceptions. It is understood that 55.2% of students found correct solutions for the physical meaning and real-life status of the slope, 18.8% made mistakes and 18.2 % had misconceptions.

DISCUSSION and CONCLUSION

In this section, the analysis results of the findings are presented severely for each sub-problem.

Results of the first sub-problem

The results reached as a result of the analyses of the findings within the scope of the first sub-problem "Do the students make mistakes related to the slope? Do they have misconceptions?" are presented in this section.

As a result of the data analysis, it was concluded that the students made some mistakes about the slope and had misconceptions (Table 1, and Table 2).

Results of the second sub-problem

The results reached as a result of the analyses of the findings within the scope of the second sub-problem "What are the mistakes and misconceptions according to Stump's Slope perception classification?" are presented below.

As a result of the data analyses, it was found that,

It is the "parametric coefficient" meaning of the slope in which the students made mistakes at the lowest rate [2,4% (Table 3], in addition, were not able to solve at the high rate [23,6% (Table 3)].

It is the "algebraic rate and parametric coefficient" meaning of the slope in which the students made mistakes at the low rate [3% (Table 3)], were not able to answer at the highest rate [57 % (Table 3)], and the correct answer rate was low [26,1 % (Table 3)].

It is the "physical meaning and daily life" meaning of the slope that students made higher mistakes [18,8% (Table 3)], and the rate of correct answering was high [55,2% (Table 3)].

It is the "trigonometric" meaning of the slope in which the students made mistakes at the highest rate [21,2% (Table 3)], and gave a low rate of correct answers [39,4% (Table 3)].

It is the "real-life situations" meaning of the slope that the students made mistake at the low rate (5,5% (Table 3)), with the highest answering rate [69,7% (Table 3)], and left empty [3,6% (Table 3)] at the lowest rate.

It was determined that the students had a different rate of misconceptions related to the "geometric meaning" and "geometric rate" of the slope (12,1% for the 1st and 12^{th} questions; 44,8% for the 6th and 11^{th} questions [Table 3)]. These results demonstrate similarity with the study by Clement (1985), Duncan and Chick (2013) suggesting that "Students who learn the slope at an operative level usually focus only on height and do not pay attention to the length of the horizontal." Some students interpreted the situation as "there is no slope" when they obtained a line parallel to the horizontal axis. This result indicates that students have problems in transferring some of the information they have gained through experiences to the conceptual level. The information "There is no ramp on the straight road" was regarded as "There is no slope" by these students and this result is parallel with the result reached by Yenilmez and Yaşa (2008) as problems can be experienced while transferring informal information to the formal level may reveal misconceptions.

It was determined that one-quarter of students had misconceptions related to the "trigonometric meaning" of the slope [23,6 % (Table 3)].

It was observed that more than one-fifth of students had misconceptions related to the "real-life situation" meaning of the slope (21,2% [Table 3)].

It was determined that approximately one-fifth of students had misconceptions related to the "parametric coefficient" meaning of the slope [17,6% (Table 3)]. This result is parallel with the result of the study by Barr (1981) as "Is the slope m or c in the equation y= mx + c?"

It was also determined that few students [13,9% (Table 3)] had misconceptions related to the "algebraic rate and parametric coefficient" meaning of the slope.

It was observed that more than one-third of students [39,4% (Table 3)] had misconceptions related to the "geometric rate and algebraic rate" meaning of the slope. This result shows similarity with the result reached by Crawford and Scott (2000) as "the students had difficulty in relating the informal and formal forms of the slope" and the result by Barr "Is the slope of the line whose two points are known division of the difference between "x" s by the difference of "y" or vice versa?"

Approximately one-fifth of students [18,8% (Table 3)] had misconceptions related to the "physical meaning and real-life situation" meanings of the slope.

Suggestions

As the results of the study, in which the mistakes and misconceptions of the students about the slope were investigated, are evaluated, the activities that can be applied in the classroom environment can be listed as follows in order to provide full learning about the slope.

1) It can be noted first of all whether we have vertical and horizontal lengths to find the slope of the hypotenuse in a given triangle.

2) Whether the slope question is given in the coordinate system or is it a real-life situation should be determined.

3) If it is the real-life situation, it can be emphasized that the result cannot be negative, and the direction of the truth is not important.

4) If the question is given in the coordinate plane, determining that it will be positive if it is leaning to the right, negative if it is leaning to the left by considering the direction of the line first and only lengths should be proportioned can be useful.

5) In finding the slope of a line with giving equation, it can be stated that it is necessary to look first at whether y is alone.

6) In a given direction (two unknowns), it can be stated that the term line in the equation does not affect the slope in any way. To do this, by changing the fixed term in the equation, a few lines can be drawn and shown that they are parallel, that is, their slopes are the same.

7) By writing a question on the board, for example, "draw the truth through the (4,3) point in the coordinate system", and all students who volunteer are given the opportunity until the answer is given correctly as that the (4,3) point is only one point and an infinite number of different truths can pass through this point.

8) To explain the straightness of the road as "slope is zero" rather than "there is no slope," draw a straight line (path) on the board and by asking questions in the form of "what do you think about the slope of this?', the answers can be discussed in the classroom environment.

9) Studies on how the study of the subject of education in out-of-school learning environments affects the conceptual learning level of the subject can be conducted.

10) Studies on how students interpret the slope in solving mathematical problems can be conducted.

11) Studies on what problems are encountered while transferring the images of slope acquired through experiences to the conceptual level can be conducted.

REFERENCES

Barr, G. (1981). Some student ideas on the concept of gradient. *Mathematics in School*. *10*(1), 14-17.

Cheng, D. S. (2010). *Connecting proportionality and slope: Middle school students' reasoning about steepness*. Boston: Boston University.

Clement, J. (1985, July). Misconceptions in graphing. In Proceedings of the ninth international conference for the psychology of mathematics education (Vol. 1, pp. 369-375). Utrecht, the Netherlands: Utrecht University.

Crawford, A. R., & Scott, W. E. (2000). Making sense of slope. *The Mathematics Teacher*, 93(2), 114-118.

Creswell, J. W., & Clark, V. L. P. (2007). *Designing and conducting mixed methods research*. CA, Sage publications.

Duncan, B., & Chick, H. L. (2013). How do adults perceive, analyse and measure slope?. *In 36th annual conference of Mathematics Education Research Group of Australasia*, 258-265.

Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989). Toward a conceptual framework for mixed-method evaluation designs. *Educational Evaluation and Policy Analysis*, 11(3), 255-274.

Hoffman, T. W. (2015). *Concept image of slope: Understanding middle school mathematics teachers' perspective through task-based interviews.* (Unpublished doctoral dissertation). The University of North Carolina at Charlotte.

Milli Eğitim Bakanlığı (MEB) (2017). *İlköğretim matematik dersi 5-8. sınıflar öğretim programı ve kılavuzu*. Ankara: Devlet Kitapları Müdürlüğü Basımevi.

Morse, J. M. (1991). Approaches to qualitative–quantitative methodological triangulation. *Nursing Research*, *40*, 120–123.

Özbellek, G. (2003). *İlköğretim 6. ve 7. sınıf düzeyindeki açı konusunda karşılaşılan kavram yangıları, eksik algılamaların tespiti* [Confronted conceptions mistakes about angle, determaniation and elimination methods of deficient perceptions at 6th and 7th grades in primary education], (Unpublished Master Dissertation). Dokuz Eylül University, İzmir.

Stump, S. (1999). Secondary mathematics teachers’ knowledge of slope. *Mathematics Education Research Journal, 11*(2), 124-144.

Styers, J. L., Nagle, C. R., & Moore-Russo, D. (2020). Teachers’ noticing of students’ slope statements: attending and interpreting. *Canadian Journal of Science, Mathematics and Technology Education, 20*(3), 504-520.

Yenilmez, K., & Yaşa, E. (2008). İlköğretim öğrencilerinin geometrideki kavram yanılgıları [Primary school students’ misconceptions about geometry]. *Journal of Uludag University Faculty of Education, 21*(2), 461-483.

Yıldırım, A., & Şimşek, H. (2016). *Sosyal bilimlerde nitel araştırma yöntemleri* [Qualitative research methods in social sciences], 9th Edition, Ankara: Seçkin Publishing 256-272.

Zandieh, M. J. (2000). A theoretical framework for analyzing student understanding of the concept of derivative. *CBMS Issues in Mathematics Education, 8,* 103-127.