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THE IMPORTANCE OF ALGEBRA TEACHING; DAILY LIFE VARIABLES AND NUMBER SYSTEMS CORRESPONDING TO THESE VARIABLES

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Abstract

The aim of the study is to determine which number systems do the mathematics teachers use to associate the daily life variables in daily life problems, and if they have associated, to find whether they used the algebraic features of the number systems they associated. In the research, the participants' ability to transform between number sets and daily life variables and their using of the features of the algebraic structure of these number sets were tried to be investigated in detail. With this purpose, the case study method, one of the qualitative research patterns, was employed in the research. The research was carried out with 15 secondary school mathematics teachers, studying at the Post-graduate Department of the Secondary School Mathematics Teaching in the 2014-2015, 2015-2016 and 2016-2017 academic years. For this reason, the study was designed on the use and awareness of knowledge. As the data collection tools, the mid-term and final exams papers were applied. The results suggest that the penny and algebraic features of number systems and the concept of "equation" are not correctly formed, therefore which number systems are the daily life variables valued in is not exactly known.

Keywords: student awareness, algebra teaching, variable, mathematics in daily life, teacher education.

INTRODUCTION

In addition to the significant features such as generalization and common signification (language), mathematics is difficult since it establishes abstract relations that are always valid at least between the infinite variable. However, the historical process has shown that mathematics is a sort of science that can be learned and taught. It is obvious that as more individuals' learning mathematics as possible contributes to the individual, the surrounding and country in which s/he live, science, thus humanity. This situation caused countries to include mathematics in almost all stages of the educational programs. For this reason, mathematics curricula were developed and institutions that would train teachers to apply these curricula were settled.

Mathematics includes consecutive steps that are interdependent and cannot absolutely be skipped. Mathematics instructors should provide awareness of those, who will get educated related to this side of the mathematics. Otherwise, as a structure other than mathematics' own structuring is created in any step of teaching, sometimes irreversible misconceptions may arise and mathematics knowledge can be perceived as "born knowledge". In the algebra, which is a sub-branch of mathematics, this situation becomes more obvious and may bring the learner in a situation that prevents them from doing mathematics.

The algebra has a significant role in forming other fields of mathematics. For this reason, the learners who are not able to completely form the algebra will even have the problems in other branches of mathematics. This situation, which contributes a particular significance to algebra, brings forth the algebra instructors and the institutions that teach these instructors, that is, mathematics teachers' structuring algebra in its own structure is gaining importance. When this is provided, instructors can use their knowledge in solving current life problems until a future predictable with the achievements of this period and make mathematical modelling. It is significant to know with which number sets the daily life variables can be expressed in mathematics to solve daily life problems. Because, the algebraic structure of that set of numbers will enable the decision about the existence and non-existence of the solution, and if there is a solution, the solution steps and the application of these steps will be a guide. Therefore, teachers knowing with which number sets the current life variables can be expressed in mathematics and applying this knowledge is significant and in this scope, the studies that will be conducted can be a reference source for institutions that educate teachers. According to the literature review, the studies related to algebra teaching can be summarized under these headings;

- *difficulties and misconceptions in algebra* (Akgün & Özdemir, 2006; Akkan at all, 2009; Akkaya & Durmuş, 2006; Akkaya, 2006; Aksu, 1997; Arzarello at all, 1993; Baki & Kartal, 2002, 2004; Barbieri at all, 2019; Barnard, 1989; Başgün & Ersoy, 2000; Birenbaum at all, 1993; Birgin & Demirören, 2020; Birgin & Gürbüz, 2009; Dede & Argün, 2003; Dede & Peker, 2007; Dede at all, 2002; Erbaş at all, 2009, 2010; Erbaş at all, 2014; Erdem & Gürbüz, 2017; Ersoy & Erbaş, 2005; Ferretti, 2020; Herscovics & Linchevski, 1994; Keşan & Akbulut, 2019; Kieran, 1992),
- *problem-solving* (Akkuş & Çakıroğlu, 2006; Bedel & Arı, 2012; Brown & Walter, 1993; Cenkseven & Akar, 2006; Demirtaş & Dönmez, 2008; Genç & Kalafat, 2007; Herscovics & Linchevski, 1994; Karataş & Güven, 2003; Küchemann, 1978; Lavy & Bershadsky, 2003; Macnair & Elliot, 1992; Moses at all, 1990; Moss & Lamberg, 2019; Pawley at all, 2005; Polat &Tümkaya, 2010; Sahal & Özdemir 2019; Saracaloğlu at all, 2001; Saracaloğlu at alll, 2009; Serin, 2006; Sun at all, 2019; Tatar & Soylu, 2006),
- *problem-posing* (Akay at all, 2006; Akkan at all, 2009; Barlow & Cates, 2006; Brown & Walter, 1993; Cai, 2003; Cankoy & Darbaz, 2010; Cristou at all, 2005; English, 1997; Fidan, 2008; Işık at all, 2011; Kaya & Keşan, 2014; Korkmaz & Gür, 2006; Lavy & Bershadsky, 2003; Mayer, 1982; Moses at all, 1993; Nardone & Lee, 2010; Nixon-Ponder, 1995; Silver & Cai,1996; Silver, 1997; Toluk-Uçar, 2009),
- *algebraic thinking* (Akkuş & Çakıroğlu, 2006; Apsari at all, 2020; Bağdat, 2013; Blanton & Kaput, 2005; Blume & Heckman, 2000; Çağdaşer, 2008; Çelik, 2007; Driscoll, 1999; Gülpek, 2006; Herbert & Brown, 1997; İspir & Palabıyık, 2011; Kaf, 2007; Kaş, 2010; Kaya & Keşan 2014; Kieran & Chalouh, 1993; Kim, 2020; Lawrence & Hennessy, 2002; Palabıyık, 2010; Powell at all, 2019; Steele & Johanning, 2004; Stewart at all, 2019; Vance, 1998; Walle at all, 2013; Wilkie, 2019; Yıldız & Akyüz, 2020),
- *associating with daily life* (Ball & Cohen, 1996; Durmuş, 2005; Litke, 2020; Stein & Henningsen, 1997; Styers at all, 2020; Stylianides & Stylianides, 2008; Wagner, 1983),
- *mathematical modelling* (Erbaş at all, 2014; Etcuban at all, 2019; Gravemeijer & Stephan, 2002; Haines & Crouch, 2001, 2007; Kertil, 2008; Lehrer & Schauble, 2003; Lesh & Doerr, 2003; Lingefjard & Holmquist, 2005; Lingefjard, 2002; 2004, 2006; Niss at all, 2007; Verschaffel & De Cote, 1997; Verschaffel at all, 2002).

Research Purpose

The aim of the study is to determine which number systems do the mathematics teachers use to associate the daily life variables in daily life problems, and if they have associated, to find whether they used the algebraic features of the number systems they associated. For this reason, the study was designed on the use and awareness of knowledge.

Research Problem

How and with which number systems do mathematics teachers associate daily life variables in writing and solving daily life problems?

METHODS

In the research, the students' ability to transform between number sets and daily life variables and their using of the features of the algebraic structure of these number sets were tried to be investigated in detail. Considering this purpose, among the qualitative research patterns, the case study method, which enables to examine a case in-depth with the expression of McMillan (2000) was employed in the research.

Study Group

The research was carried out with 15 secondary school mathematics teachers, who were studying at the Post-graduate program of Secondary School Mathematics Teaching. All of the participants were those who graduated from the mathematics education department and took the relevant courses on number sets and algebraic structures of these sets and succeeded in these courses. All of the were officially on duty during the research process. For this reason, the study was designed on the use and awareness of knowledge.

Data Collection Tools

The mid-term and final exam papers of the Algebra Teaching I and II, which are taught at the Post-graduate Program of the Secondary School Mathematics Teaching were used as the data collection tools. In order to investigate where the number sets used in algebra teaching can correspond to daily life, the question.
Question: In accordance with the x+3=7 equation, the following question was asked to the first and second group, as write

a) A Natural number,

b) An Integer,

c) A Rational number,

d) A Real number

The problem from daily life.

As the data gathered from the first group demonstrated that there are difficulties in terms of what the number sets can correspond to in the daily life, the awareness of the algebraic structures of the number sets that are important in the solution of the problem was intended to be determined. For this reason, the second group was also asked to solve the problems that they wrote to examine the awareness of the algebraic features of the sets they used in the solution.

Collection and Analysis of the Data

The duration for the participants to answer the questions one week. The students obeyed the exam rules during the answering process. The exam papers taken from each student were coded from E-1 to E-6 for the first group and from F-1 to F-9 for the second group and transferred to the computer environment. The analysis of the gathered data was done under the headings of "Writing a Natural Number Problem", " Writing an Integer Problem", "Writing a Rational Number Problem", "Writing a Real Number Problem" for the first group appropriate to the x+3=7 equation. The data gathered for each heading were subjected to the descriptive analysis which is used during the cases in which the conceptual structure has already known (Yıldırım & Şimşek, 2008, p. 224), considering the codes "D for correct", "Y for incorrect" and "B for not answered" created by two academicians who were the professionals in algebra and qualitative research. For the second group, additionally, suitable for the equation x+3=7, the evaluation was done under the headings of "Solving Correct", "Solving with Missing Step", "Solving Incorrect", "Leaving Empty". All of the answers by the participants were taken as they were and evaluated by specifying causality under these headings.

RESULTS

The answers of the participants to the research questions are presented with tables according to their categories. In addition, the answers of each participant were analyses one by one and detailed evaluation was done.

The distribution of the participants answers to the question of writing a daily life problem consisting of "Natural number, Integer, Rational number and Real number" variables, appropriate to the x+3=7 equation, according to the categories of "D for correct, Y for Incorrect and B for Empty" categories is presented in Table 1.

Table 1. The Distribution of the Participants Answers to the Question of Writing a Daily Life Problem Consisting of "Natural Number, Integer, Rational Number and Real Number" Variables, Appropriate to the x+3=7 Equation, According to the Categories of "D for Correct, Y for Incorrect and B for Empty" Categories.

Participant and Number	Group	Suitable for the x+3=7 Equation											
		Writing a Natural Number Problem			Writing an Integer Problem			Writing a Rational Number Problem			Writing a Real Number Problem		
1st Group		D	Y	B	D	Y	B	D	Y	B	D	Y	B
1		D					B		Y				B
2				B		Y			Y			Y	
3		D				Y			Y				B
4		D				Y				B		Y	
5		D				Y				B			B
6		D				Y				B			B
	Total	5	0	1	0	5	1	0	3	3	0	2	4
2nd Group		D	Y	B	D	Y	B	D	Y	B	D	Y	B
1		D			D				Y			Y	
2		D			D				Y		D		
3			Y			Y			Y				B
4		D				Y			Y			Y	
5		D				Y			Y		D		
6			Y			Y			Y			Y	
7			Y			Y			Y			Y	
8		D				Y			Y			Y	
9		D				Y			Y				B
	Total	6	3	0	2	7	0	0	9	0	2	5	2

The distribution of the groups' answers to the question of writing a daily life problem consisting of "Natural number, Integer, Rational number and Real number" variables, appropriate to the x+3=7 equation, according to the categories of "D for correct, Y for Incorrect and B for Empty" categories is presented in Table 2.

Table 2. The Distribution of the Groups' Answers to the Question of Writing a Daily Life Problem Consisting of "Natural Number, Integer, Rational Number and Real Number" Variables, Appropriate to the x+3=7 Equation, According to the Categories of "d for Correct, y for Incorrect and b for Empty" Categories.

Participant Group and Number	Suitable for the x+3=7 Equation											
	Writing a Natural Number Problem			Writing an Integer Problem			Writing a Rational Number Problem			Writing a Real Number Problem		
	D	Y	B	D	Y	B	D	Y	B	D	Y	B

1st Group	5	0	1	0	5	1	0	3	3	0	2	4
2nd Group	6	3	0	2	7	0	0	9	0	2	5	2
Total	11	3	1	2	12	1	0	12	3	2	7	6
General Total		15			15			15			15	

The distribution of the participants' answers to the question of writing a daily life problem consisting of "Natural number, Integer, Rational number and Real number" variables, appropriate to the x+3=7 equation, according to the categories of "Solving Correct", "Solving with Missing Step", "Solving Incorrect", "Leaving Empty" categories is presented in Table 3.

Table 3. The Distribution of the Participants' Answers to the Question of Writing a Daily Life Problem Consisting of "Natural Number, Integer, Rational Number and Real Number" Variables, Appropriate to the x+3=7 Equation, According to the Categories of "Solving Correct", "Solving with Missing Step", "Solving Incorrect", "Leaving Empty" Categories.

Written appropriate to the x+3=7 equation	Solving Correct	Solving with Missing Step	Solving Incorrect	Leaving Empty	Total
Natural Number Problem	0	3	5	1	9
Integer Problem	1	6	0	2	9
Rational Number Problem	0	5	0	4	9
Real Number Problem	1	4	0	4	9
Total	2	18	5	11	36

As it is seen in Table 1,2 and 3, while the highest number of correct answers occurs at "Natural numbers" (11 correct answers), there is no correct answer in "Rational numbers", also, the number of correct answers in "Real numbers and Integers" is the same and is 2. The number of incorrect answers which is most in problem writing is in "Rational and Integers (12 incorrect answers) and the most number of questions left empty is in "Real numbers". While the correct answer in problem-solving is in "Integers and Real numbers" (1 correct solution), there is no correct solution in "Natural and Rational numbers". The solution with missing step occurred in "Integers" most (6) and least is in "Natural numbers" (3). The highest number of incorrect answers occurs in "Natural numbers" (5) and there is no incorrect solution among others. While leaving empty in "Rational and Real numbers" is the same (4), it is 2 in "Integers" and 1 in "Natural numbers."

The answers of six participants stated in the first group were taken as they were (without correcting the missing sentences and miswriting) and the evaluations for the answers are given just below the answer.

The answers of E1 coded participant and evaluations for the answers are presented below.

a) Ali adds 3 more to his chocolates. As he has 7 chocolates now, how many bars of chocolates did he have at the beginning? This is a natural number problem. Because the result can only be "0" and positive numbers.

Evaluation: The answer of the participant was evaluated in the category of 'correct'. However, the expression stated in the explanation of the answer "Because the result can only be "0" and positive numbers" is far from mentioning that the problem is a natural number problem. Besides, mentioning positive numbers indicates that the difference between natural numbers and integers is unknown.

b) Ali takes 3 slices of a cake. He has 7 slices in total now. How many slices did he have in the beginning? As the slice is a part of a cake, it can be written as a/b. That is, it is a fractional expression. So, it is a rational number problem.

Evaluation: The answer of the participant took place in the category of 'incorrect'. Because the mathematical writing of the problem stated by the participant is not "x+3=7". In addition, how many slices were the cake divided was not stated in the problem? The expression of the participants "That is, it is a fractional expression. So it is a rational number" statement indicates that the participant has some insufficient knowledge in the concepts of fractional numbers.

c) No answer

d) No answer

The answers of E2 coded participant and evaluations for the answers are presented below.

a) No answer

b) When Ali's father gave him 3 apples, he has 7 apples now. In this case, how many apples did Ali have at the beginning? This is an integer problem. Because 3 is a natural number and 7 is an integer. In this case, 7 minus 3 equals to an integer too.

Evaluation: The answer of the participant stated in the category if 'incorrect' Because the expression in the explanation of the answer "This is an integer problem. Because 3 is a natural number and 7 is an integer. In this case, 7 minus 3 equals to an integer too" is far from stating that it is an integer problem.
This refers that the difference between natural numbers and integers are not known.

c) Ali has 3 halves of an apple. His father gave him 4 apples. How many apples does Ali have in total? This is a Rational number problem. Because we use Rational numbers to express with the half apple.

Evaluation: The answer of the participant was evaluated under the category of 'incorrect'. Because of mathematical expression that the participant wrote is not "x+3=7". In addition, the expression of the participant as "Because we use Rational numbers to express with the half apple." indicates insufficient knowledge in rational numbers, fractional number concepts.

d) If Ali waits for Ayşe for 3 hours, it is 4.

Evaluation: As the answer of the participant does not include a sentence, it was evaluated under the category of 'empty'.

The answers of E3 coded participant and evaluations for the answers are presented below.

a) There are some students in a classroom. If 3 more students are included in this classroom, there will be 7 students in total. How many students were there in the beginning?

Evaluation: The answer of the participant stated in the category of 'correct'.

b) x+5=3; x+3=7
Ayşe has got 3 Liras. However, she owes 5 Liras. How much money does she has after paying her debt? (The debt is expressed with (-)) If we write the same example even for X+3=7, x+3=7 will be an integer problem.

Evaluation: As the indirect mathematical expression of the participant is not "x+3=7", the answer of the participant was evaluated under the category of 'incorrect'.

c) x+3=7
Fatma has got 6 halves of an apple. How many more whole apples should her mother give her to have 7 whole apples?

Evaluation: The answer of the participant was evaluated under the category of 'incorrect'. Because the mathematical expression of the problem that the participant stated is not "x+3=7".

d) No answer

The answers of E4 coded participant and evaluations for the answers are presented below.

a) As 3 more of Ali's marbles is 7, haw many marbles does Ali have. (Natural number)

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

b) Which is the number that we add 3 and get 7? (Integer)

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

c) No answer

d) The solution set of x+3=7 equation (Real number)

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

The answers of E5 coded participant and evaluations for the answers are presented below.

a) Ali has got some money in his pocket. When he took 3 Liras from Mehmet and his money is 7 Liras now. How much money did he have at the beginning? A natural number problem.

Evaluation: The answer of the participant was evaluated under the category of 'correct'. However, the expression of the participant " a natural number equation" indicates that the participant could not comprehend the relationship between equation and problem.

b) Mehmet climbs 3 meters higher than he is. The height is 7 meter now. What was the height at the beginning? An integer problem.

Evaluation: The participant's answer was evaluated under the category of 'incorrect'. Because height and length are expressed in real numbers. This situation shows that the participant could not comprehend the concepts of real numbers and integers.

c) No answer

d) No answer

The answers of E6 coded participant and evaluations for the answers are presented below.

a) An integer problem: There are 3 students in a class that should have 7 students. According to this, how many students are needed?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

b) An integer problem: 3 meters of a 7-meter road is muddy. According to this, how many meters is the length of the road without mud?

Evaluation: The participant's answer was evaluated under the category of 'incorrect'. Because the distance is expressed with real numbers. This case indicates that the participant could not comprehend the concepts of the real number and integer properly.

c) No answer

d) No answer

The answers of six participants stated in the second group were taken as they were (without correcting the missing sentences and miswriting) and the evaluations for the answers are given just below the answer.

The answers of F1 coded participant and evaluations for the answers are presented below.

a) "There are 3 people in an elevator. How many more people get on this elevator, the total number of people in the elevator will be 7?"

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

b) "Find out how many steps Ali is away from the water well, which he needs to take 3 more steps to reach the water well 7 steps away."

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: 7-3= 4 steps

Solution Evaluation: Participant's solution is the same for natural and integers and does not include solution steps.

For instance, Since there is not the opposite of 3 in the addition operation for natural numbers, the solution of x+3=7 is x+3=7=4+3 and as shortened, it is as x=4;

3 has the opposite for the addition and it is (-3) for integers, rational and real numbers

The solution of x+3=7 is x+3=7, as x+3+(-3)=7+(-3) is x+ [(+3)+(-3)]=+4 is x+0=+4 , that is, x=+4 .

Even this case was not regarded as appropriate for the natural numbers. The solution is taken as a missing step solution for integers.

c) “Find the number a if x + 3 = 7 with x equal to 3.a / 2”

Evaluation: As the answer of the participant did not include a daily life problem, the answer of the participant was evaluated under the category of ‘incorrect’.

Solution: Since the value of x in the expression x + 3 = 7 is 4, the number a becomes 8/3, which is one-third of 2 times 4.

Solution Evaluation: As the solution was obtained without using any processing step where the value of "x" is 4, the solution was evaluated under the category of ‘incorrect’.

d) As “X is the square of “b” natural number, if x+3=7, find “b”.”

Evaluation: As the answer of the participant did not include a daily life problem, the answer of the participant was evaluated under the category of ‘incorrect’.

Solution: As $b^2=4$, $b=\sqrt{4}=2$.

Solution Evaluation: The participant took the “x” value as 4 without operation. Therefore, the solution was evaluated as with missing step.

The answers of F2 coded participant and evaluations for the answers are presented below.

a) As I add 3 more apples to my apples, the number of the apples is 7, how many apples did I have at the beginning?

Evaluation: The answer of the participant was evaluated under the category of ‘correct’.

Solution: 7-3=4

Solution Evaluation: The solution was not considered as appropriate to natural numbers.

b) Which floor was the person at the beginning as s/he goes up 3 floors with the elevator, comes to the 7th floor?

Evaluation: The answer of the participant was evaluated under the category of ‘correct’.

7-3=4

Solution Evaluation: Solution was evaluated under missing steps as there were missing digits

c) When Ayşe had 3 liras added to her money, she had 7 liras, how much money did she had at the beginning?

Evaluation: As the data in the answer of the participant was expressed with rational numbers, the answer of the participant was evaluated under the category of ‘incorrect’

Solution: 7-3=4

Solution Evaluation: The solution was evaluated as missing digits in rational numbers.

d) If Hasan buys a $3m^2$ land next to the land inherited from his father, his land becomes $7m^2$. How many m^2 is the land inherited from Hasan's father?

Evaluation: The answer of the participant was evaluated under the category of ‘correct’.

Solution: 7-3=4

Solution Evaluation: The solution was evaluated as a missing step in real numbers.

The answers of F3 coded participant and evaluations for the answers are presented below.

a) Ahmet is 3 years older than Ali. As Ahmet is 7 years old, how old is Ali?

Evaluation: As the data in the answer of the participant was expressed with real numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: x= Ali's age, X+3=7 x=4

Solution Evaluation: The solution was evaluated as a missing step in natural numbers.

b) When a friend of him gave 3 marbles to Murat, who had already had some marbles, he had 7 marbles. How many marbles did he have at the beginning?

Evaluation: As the data in the answer of the participant was expressed with natural numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: X= Murat's money at the beginning

X+3=7 x=4

Solution Evaluation: The solution was evaluated as a missing step in integers.

c) Rational number problem: Ali bought a cake on his birthday. He cut his cake into 7 pieces. He shared 3 slices of these seven pieces for those in the house. Ali gave the rest of the cake for his friends. How many slices of the cake did Ali share with his friends?

Evaluation: As the indirect mathematical expression of the participant is not "x+3=7", the answer of the participant was evaluated under the category of 'incorrect'.

Solution Evaluation: As there was no solution, the evaluation was not done.

d) I couldn't write.

The answers of F4 coded participant and evaluations for the answers are presented below.

a) If Ayşe, that we do not know how many bags she has, buys 3 bags and reaches 7 bags, how many bags has she got before?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: The expression of bag refers to a natural number. It refers to a natural number, as it can not be a minus or fractional bag. As we do not know how many bags she has, if we think as if she did not take the 3 bags she bought and come back 3 numbers from 7, we reach 4. As 3+4=7, she must have 4 bags that if she buys 3 more, she can reach 7.

Solution Evaluation: Although the oral expression of the solution is correct, as the mathematical expression of this verbal statement does not contain steps to show awareness of algebraic structure, the solution was regarded as with missing digits.

b) As the temperature increases 3 degrees in an uncertain day, it reaches 7 degrees at noon, what was the temperature at the beginning?

Evaluation: As the data in the answer of the participant was expressed with real numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: Air temperature expression refers to an integer. Because temperature can be positive negative. Since it is an integer, increasing 3 degrees means +3. Being 7 degrees means being +7. If it increases to 7 degrees with 3 degrees more at an unknown temperature, we can solve the question by thinking as follows. How much increase, if we put over 3 degrees, is 7 degrees? It will be a positive number because there is an increase. To get from 3 to 7 degrees, it takes 4 more degrees of increase. That is the temperature at the beginning was 4 degrees.

Solution Evaluation: The expression in the solution "Air temperature expression refers to an integer" is a common fault. Because, as the temperature is changeable and can be referred to with real numbers. That is, the participant built the solution on a different number system. Although the verbal statement of the solution can be considered correct in integers where the participant built the solution, the solution was regarded as with the missing steps as the mathematical expression of this verbal statement does not include steps to show awareness of algebraic structure.

c) Ahmet, who bought 3 loaves of bread from the grocery, he saw that there were 7 loaves of bread at home. How many loaves of bread were there at home at the beginning?

Evaluation: As the data in the answer of the participant was expressed with natural numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: The expression of bread refers to a rational number because there can be half bread and a quarter bread etc. If he buys 3, he reaches 7, we should think of that. Let's imagine the situation before buying 3 loaves and subtract 3 from 7. Since there were 4 left, it means there were 4 loaves of bread.

Solution Evaluation: The statement in the solution "The expression of bread refers a rational number" is not specified, it is not possible to say anything about its true or false, but not mentioning the same parts in the following statements shows that it is expressed with natural numbers. Besides, the participant built the solution on a different number system. Although the verbal statement of the solution can be considered correct in rational numbers that the participant has designed the solution, the solution was considered with missing steps, since the mathematical expression of this verbal statement does not consist of steps to show the awareness of the algebraic structure.

c) Which is the number that we add 3 and get 7?

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

Solution: Which number says the problem, so we don't know the number. If we say x to the number, the problem equation is set up as follows.

$x+3=7$.

Here the x expression refers to a real number. We can solve the equation like this. We try to find the unknown, that is we need to find x. The +3 statement next to it should be omitted The number that will destroy +3, add zero, is -3. If we add -3 to both sides of the equation, the equality will not be broken, so let's apply it now.

$$(-3) + x + 3 = 7 + (-3) \qquad x=4$$

Solution Evaluation: The participant's expression "Here the x expression refers to a real number" indicates that the participant could not comprehend the concept of "equation" properly as it was taken with the problem statement. However, as the solution of the participant was analysed, it is understood that s/he is aware of the algebraic structure s/he uses, except for the deficiencies such as not using the combination feature with the right-left operation.

The answers of F5 coded participant and evaluations for the answers are presented below.

a) Elif has 7 pencils as her friend gives 7 pencils to her, Accordingly how many pencils did she have at the beginning?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: x: the number of pencils Elif had at the beginning

$X+3=7$

$X+3-3= 7-3$

$X= 4$

Solution Evaluation: As the evaluation of the participant was analysed, it was understood that the participant used the algebraic features of integer, rational and real numbers instead of the algebraic features of natural numbers in the solution. For this reason, the solution was regarded as incorrect.

b) Elif has 7 pencils as her friend gives 7 pencils to her, Accordingly how many pencils did she have at the beginning?

Evaluation: As the data in the answer of the participant was expressed with real numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: x: the number of pencils Elif had at the beginning
x+3=7
x+3-3= 7-3
x= 4

Solution Evaluation: As the solution of the participant is analysed, although it is not clearly understood that the opposite of +3 is -3 according to the addition operation (if x + [3 + (- 3)] = 7 + (-3) it would be clearly understood that the opposite of +3 is -3), it is seen that the participant is aware of the algebraic features of integers. For this reason, the solution was regarded as correct.

c) In the first month of the birth, Ayşegül, after taking 3 kg, reached to 7 kg. Accordingly, what was her weight when she was born?

Evaluation: As the data in the answer of the participant was expressed with real numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: No answer

Solution Evaluation: As there was no solution, evaluation could not be done.

d) Real number problem:

- If a person, who wants to fence one side of a square-shaped field with an area of 49 m2, has a fence of 3 m in his hand, how many meters more fences does he need to make entire area fenced?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: No answer

Solution Evaluation: As there was no solution, evaluation could not be done.

The answers of F6 coded participant and evaluations for the answers are presented below.

a) A tree was planted. It extends 3 meters in a year and the height reaches at 7 meters. What was the height of it at the beginning?

Evaluation: As the data in the answer of the participant was expressed with real numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: No answer

Solution Evaluation: As there was no solution, evaluation could not be done.

b) A child is given 3 liras by his father. The child has 7 liras now. How much money did he have at the beginning?

Evaluation: As the data in the answer of the participant was expressed with natural numbers, the answer of the participant was evaluated under the category of 'incorrect'.

Solution: No answer

Solution Evaluation: As there was no solution, evaluation could not be done.

c) Ali bought a cake on his birthday. He cut his cake into 7 pieces. He shared 3 slices of these seven pieces for those in the house. Ali gave the rest of the cake for his friends. How many slices of the cake did Ali share with his friends?

Evaluation: As the mathematical writing in the answer of the participant was not "x+3=7", the participant's answer was evaluated under the category of 'incorrect'.

Solution: No answer

Solution Evaluation: As there was no solution, evaluation could not be done.

d) The problems above can be evaluated under the real number.

Evaluation: The participant stated that her/his answer as a rational number problem would be taken as a real number problem. Accordingly, since the mathematical expression of the problem in his/her answer was not "x + 3 = 7", the participant's answer was included in the category of incorrect.

In addition, the participant also has a misconception that every rational number problem can be taken as a real number problem.

The answers of F7 coded participant and evaluations for the answers are presented below.

a) Ali has got 7 marbles. Veli's marbles are 3 fewer than Ali's. Accordingly, how many marbles do Veli have?

Evaluation: As the mathematical writing in the answer of the participant was not "x+3=7", the participant's answer was evaluated under the category of 'incorrect'.

Solution: Ali's marbles = 7   Veli's marbles = x,

X+3=7 that is,   x=4. Veli has got 4 marbles.

Solution Evaluation: Although the result in the solution was correct since the solution steps were not stated, the solution was regarded under the category of the solution with missing steps.

b) Every day a trader sells a commodity that is less than the previous day. He sold a total of 7 pieces of goods in two days. How many pieces of goods did he sell on the first day?

Evaluation: As the mathematical writing in the answer of the participant was not "x+3=7", the participant's answer was evaluated under the category of 'incorrect'.

Solution:

1st day: x

2nd day: +   x-1

Total: 7 = 2x-1. 2x=8, x=4 and x-1=3. From that point x+3=7

Solution Evaluation: The participant solved his own question. Although the result was correct, as there were no steps, the participant's solution was regarded as the solution with missing steps.

c) 1/5 of a number plus 3 is 7. What is this number?

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

Solution:

If we refer 1/5 of the number as x,   x+3=7. from that point x=4. 4×5=20, the number is 20.

Solution Evaluation: The participant solves his/her own question. Although the result was correct, the solution of the participant was evaluated under the category of 'solution with missing step' as there were not the steps in the solution.

d) X is 8 times a real number and x + 3 = 7. What is this number?

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

Solution: x+3=7 and   x=4, number=4/8=1/2

Solution Evaluation: The participant solves his/her own question. Although the result was correct, the solution of the participant was evaluated under the category of 'solution with missing step' as there were not the steps in the solution.

The answers of F8 coded participant and evaluations for the answers are presented below.

a) Ali has got 3 pencils. How many pencils should he get to reach 7?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: As Ali had 3 pencils at the beginning and 7 afterwards, the first addend is added to the second addend gives summand. The subtrahend, subtracted from the minuend, gives difference. 7-3=4 that is 4 pencils.

Solution Evaluation: Although the statement of the participant in his expression for the solution was correct, it is not the algebraic solution of the mathematical expression   "x+3=7". For this reason, the way of solution is incorrect even though the result is correct. This contradictory situation is a situation that needs to be severely investigated.

b) When Mehmet looks at the thermometer, he sees the air temperature as 3 degrees above zero. When he looks again after a while, he sees it as 7 degrees above zero, how many degrees has the air temperature increased?

Evaluation: As the mathematical writing in the answer of the participant was not "x+3=7", the participant's answer was evaluated under the category of 'incorrect'.

Solution: While the air temperature was 3 degrees above zero at the beginning, it increased slightly and became 7 degrees above zero. If we refer x to the amount of increase; 3+x=7   x=7-3   x=4 that is, it increased 4 degrees.

Solution Evaluation: Even if the participant's solution contains the correct result, it cannot be clearly seen that the opposite of +3 is -3 in the algebraic sense. In addition, the solution was taken as missing steps since it contains missing steps such as the right-left addition combination feature and unit item to reach the result.

c) Which is the number that we add 3 and get 7?

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of 'incorrect'.

Solution: Let unknown number be x , accordingly x+3=7   x=4

Solution Evaluation: Although the participant's solution included the correct result, the solution way was taken as missing steps because the steps leading to the result were missing.

d) Ayşe walked √9 meters of a √49-meter road. Accordingly, what is the meter of the rest of the road in which Ayşe walk?

Evaluation: As the mathematical writing in the answer of the participant was not "x+3=7", the participant's answer was evaluated under the category of 'incorrect'.

Solution: If we say the rest of the road x, we set the equation as;

$x+\sqrt{9}=\sqrt{49}$ $x+3=7$ $x=7-3$ $x=4$

Solution Evaluation: The participant wrote a problem that does not overlap with the expression of "x+3=7" but includes this expression. Although the result is correct both in the solution of his mathematical sentence of the problem and in the solution of "x+3=7", the result was evaluated under the category of incorrect as the steps were missing.

The answers of F9 coded participant and evaluations for the answers are presented below.

a) If Umut had 3 more toys, Umut would have 7 toys in total. Accordingly, how many toys did Umut have in the beginning?

Evaluation: The answer of the participant was evaluated under the category of 'correct'.

Solution: No answer

Solution Evaluation: Evaluation could not be done as there was no solution.

In the figure below, an ant at point A walks to reach the food at point B. As it goes from point A to point B in 3 steps, what is the number corresponding to point A at the beginning?

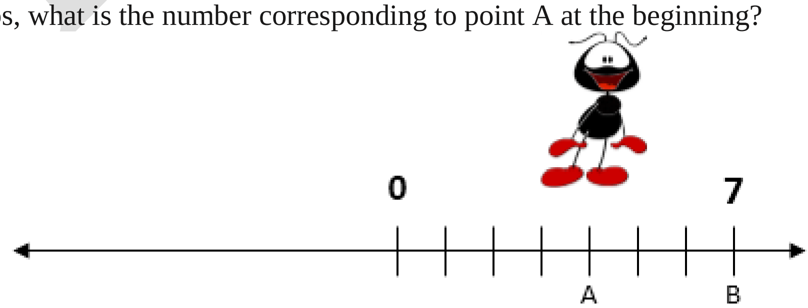

Figure 1. Figural Representation of the Problem

Evaluation: As the problem in the answer of the participant was not a real-life problem, the participant's answer was evaluated under the category of ‘incorrect’.

Solution: No answer

Solution Evaluation: Evaluation could not be done as there was no solution.

b) If three more than a number is 7, what is that number?

Evaluation: As the problem in the answer of the participant was not a real-life problem, the respondent's answer was evaluated under the category of ‘incorrect’.

Solution: No answer

Solution Evaluation: Evaluation could not be done as there was no solution.

NOTE: Sir. I understood the x + 3 =7 expression that you meant the writing problem with integers, natural numbers and rational numbers as, we need to reach the same result in all of the situations, but we need to write questions considering which set does the solutions fall into and I wrote considering this.

No answer

Evaluation: Evaluation could not be done as there was no expression.

Solution: No answer

Solution Evaluation: Evaluation could not be done as there was no solution.

DISCUSSION and CONCLUSIONS

The mathematical expression given to the participants in both groups is a fairly simple expression for the education they received and the position they were in as a mathematics teacher. On the contrary, the data are quite surprising. The participant demonstrated the highest achievement in "natural numbers" among the number systems under the heading of problem writing, but the lowest achievement (0 correct) among number systems of "integer and real numbers". The number of the correct solution in problem-solving came forth in only "real numbers and integers" and it was "1". This may be related to the thought of the participants "Why this number system?" and they do not know the answer to this question; which is actually the underlying problem. As the F9 coded participant, who stated a note as "Sir, I understood the x + 3 =7 expression that you meant as the writing problem with integers, natural numbers and rational numbers, that we need to reach the same result in all of the situations, but we need to write questions considering which set does the solutions fall into and I wrote considering this.” This is the sentence that explains this situation. It is understood from this sentence that the penny and algebraic features of number systems and the concept of "equation" are not structured correctly, therefore it is not known which number systems the current life variables are valued in.

Should the number system to be used in solving daily life problems (used in the solution) to the variables in the problem and the algebraic structure of this number system be ignored? If the number system and the algebraic structure of this number system are to be ignored, what level should it be? That is after the problem is expressed mathematically, should the solution be done and checked, ignoring the algebraic structure and number system, to see if the desired result is found? In the education of mathematics teachers, it is recommended to look for answers to these questions and to plan education according to the result. Problem-solving and problem writing take place in the first places in use by other branches of science except for mathematics teaching itself. Therefore, this approach is suitable until high school education and not suitable for education after this level.

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